Math 24
Winter 2017
Special Assignment due Monday, February 27

Let V be any n-dimensional vector space over F and W be a k-dimensional subspace of V. We know that V/W is a vector space, and that $T(x) = x + W$ is a linear transformation from V to V/W.

Assignment: Let $\alpha = \{\vec{w}_1, \ldots, \vec{w}_k\}$ be a basis for W that is contained in a basis $\beta = \{\vec{w}_1, \ldots, \vec{w}_k, \vec{v}_{k+1}, \ldots, \vec{v}_n\}$ for V. Show that if $\gamma = \{\vec{v}_{k+1} + W, \ldots, \vec{v}_n + W\}$, then γ is a basis for V/W.

Hint: To prove this directly, you need to show two things: γ spans V/W, and γ is linearly independent. You can do this by using the facts that β spans V, and β is linearly independent. For linear independence, you can note that if

$$a_{k+1}(\vec{v}_{k+1} + W) + \cdots + a_n(\vec{v}_n + W) = 0_{V/W},$$

that means that

$$(a_{k+1}\vec{v}_{k+1} + \cdots a_n\vec{v}_n) + W = \vec{0} + W,$$

and we know from earlier assignments under what conditions we have $\vec{x} + W = \vec{y} + W$.

As an alternative, you can simplify this proof by using the results of the previous special assignment.