What do we know?

Axioms for a vector space
Examples: $\mathbb{R}^n$, $\mathcal{P}_n(\mathbb{R})$, $\mathcal{P}(\mathbb{R})$, $\mathrm{Mat}_{n,m}$, $\mathbb{C}$, functions of various kinds

Linear combinations: vectors put together with addition and scalar multiplication
$\mathcal{L}(E)$ = span of $E \subseteq V$ = set of all possible linear combinations of vectors in E
$\mathcal{L}(\emptyset) = \{\mathbf{0}\}$
Subspaces: subsets that are vector spaces. $\mathcal{L}(E)$ for $E \subseteq V$.
Can show axioms hold or show the set is nonempty and closed under $+$ and $\cdot$

$E \subseteq F \ \rightarrow \ \mathcal{L}(E) \subseteq \mathcal{L}(F)$
$\mathcal{L}(E) \cap \mathcal{L}(F)$ always subspace
$\mathcal{L}(E) \cup \mathcal{L}(F)$ only subspace if it equals $\mathcal{L}(E)$ or $\mathcal{L}(F)$
need $\mathcal{L}(E) + \mathcal{L}(F) = \mathcal{L}(E \cup F)$ = sums of elements from $\mathcal{L}(E)$ and $\mathcal{L}(F)$

linear dependence for E: nontrivial linear combination from E gives $\mathbf{0}$
equivalently one vector from E obtained as a linear combination of the rest.
equivalently one vector from E obtained as a lin. comb. of the *previous* vectors.

$\dim(V)$ = size of basis for V
If $E \subseteq V$ is linearly independent, $F \subseteq V$ is spanning, then $|E| \leq \dim(V) \leq |F|$
There is a basis $\supseteq E$ (add vectors outside the span, one by one)
There is a basis $\subseteq F$ (remove redundant vectors, one by one)
If $G \subseteq V$ has size $\dim(V)$, then G is linearly independent iff $\mathcal{L}(G) = V$
$\mathcal{L}(E)$ for $E \subseteq V$ has dimension $\leq \dim(V)$; if equal, $\mathcal{L}(E) = V$.

How to Prove

$A \subseteq B$: let $x \in A$ be arbitrary, show $x \in B$
$A = B$ (sets): $A \subseteq B$ and $B \subseteq A$
$P \rightarrow Q$: assume P, show Q based on that assumption
OR assume not-Q, show not-P based on that assumption (ie show $\neg Q \rightarrow \neg P$)
$P \leftrightarrow Q$: show $P \rightarrow Q$ and $Q \rightarrow P$