Math 24 Midterm Study Guide

Material beyond the review sheets for quizzes 1 and 2:

Matrix arithmetic: addition, scalar multiplication, matrix multiplication.

Relationship between matrices and linear transformations $\mathbb{R}^n \to \mathbb{R}^m$ and how to translate between them.

Correspondence between linear combinations and compositions of linear transformations and matrix arithmetic.

Notes on linearity:

A linear polynomial is one that may be in several variables, but in which each variable is raised to the power 1 (or 0) and no distinct variables are multiplied together. A linear combination is essentially that, but with vectors playing the role of the variables (and, of course, without any constant term). Linear dependence and independence have to do with one's ability to obtain a vector of a set as a linear combination of the rest (with the exception, of course, of the linearly dependent set $\{\mathbf{0}\}$).

In all the rest of our uses, "linear" as an adjective means "well behaved with respect to linear combinations." A linear transformation is a map that preserves the relationship between vectors and linear combinations of vectors; a linear subspace (or vector subspace, though both terms are slightly redundant to just "subspace") is a subset that is closed under linear combination. Taking the linear span (also just "span") makes a linear subspace out of a subset, and taking the linear extension makes a linear transformation out of a partial map.

After two quizzes you have some sense of what shape problems in linear algebra have. How can you study for the midterm?

- Make sure you know the definitions cold. When trying to prove a new statement it is frequently most useful to start by unpacking the definitions: what exactly is being asked and what do the hypotheses tell me?
- Keep special examples in the front of your mind: when you are asked about scalars generally, consider the case 0. 2 and -1 may also be useful. When you are asked about vector spaces generally, think of $\{\mathbf{0}\}$. That is also a good example of a linearly dependent set. About linear transformations, the zero transformation and the identity, and the possibility (if allowed by the problem) of the transformation being between vector spaces of different dimensions. When asked about matrices, think of the zero matrix and the identity matrix. Remember that any nonzero vector by itself is a linearly independent set, and the spanning sets for V include V itself.
- Chain together equivalent conditions in your mind (and, in preparation for the test, on paper): for example, for injectivity of linear transformations. By definition, this means they take no two vectors to the same image. We also proved this is equivalent to taking no nonzero vector to the zero vector (i.e., having kernel $\{\mathbf{0}\}$), and to taking any linearly independent set to a linearly independent set. If furthermore the transformation is between two vector spaces of the same finite dimension, injectivity is equivalent to surjectivity.