Math 24
Winter 2010
Special Assignment due Monday, February 22

Let V be any vector space over F and W be a subspace of V. For any vector x in V, we defined the *coset* of W containing x to be

$$x + W = \{x + w \mid w \in W\}.$$

We denote the collection of cosets of W in V by V/W.

It turns out that V/W forms a vector space over F, with operations defined by

$(x + W) + (y + W) = (x + y) + W$

$a(x + W) = (ax) + W$.

You may assume that this is true. (You proved part of this in the last two special homework assignments.)

Assignment: We can define a function T from V to V/W by $T(x) = x + W$.

Prove that T is a linear transformation.

Identify the null space and range of T.

If V is finite-dimensional, what can you conclude about the dimensions of V, W, and V/W?