Math 24
Winter 2010
Special Assignment due Monday, January 11

Please hand in this assignment on a different piece of paper than the regular homework. (Be sure your name is on all assignments.)

Assignment: Let V and W be any two vector spaces over the real numbers. Define a new vector space $V \oplus W$ as follows: The elements of $V \oplus W$ are pairs (v, w) where $v \in V$ and $w \in W$. Addition and multiplication by real numbers are defined coordinatewise. This means that for any $(v, w) \in V \oplus W$, $(v', w') \in V \oplus W$, and $a \in \mathbb{R}$, we have

$$(v, w) + (v', w') = (v + v',\ w + w') \qquad a(v, w) = (av, aw),$$

where the addition and scalar multiplication in the left coordinate use the operations in V, and in the right coordinate use the operations in W.

Prove that $V \oplus W$ satisfies the following vector space axioms:

(VS 1) For all x and y in $V \oplus W$, $x + y = y + x$. (Addition is commutative.)

(VS 3) There is an element 0 in $V \oplus W$ such that for all x in $V \oplus W$, $x + 0 = x$. (The element 0 is an additive identity.)

(VS 4) For every element x in $V \oplus W$, there is an element $-x$ in $V \oplus W$ such that $x + (-x) = 0$. (The element $-x$ is an additive inverse for x.)

(VS 5) For all x in $V \oplus W$, $1x = x$.

In fact, $V \oplus W$ satisfies all the vector space axioms. Your assignment is to prove these four. Remember that this assignment is about good writing.