Math 24
Winter 2010
Quiz 9

1. A basis for the subspace $W \subseteq \mathbb{C}^3$ is $\beta = \{(1, i, 0)\}$. Find a basis for $W^\perp$.

   $W^\perp = \{v \mid \langle v, w \rangle = 0 \text{ for all } w \in W\}$, but because W consists only of multiples of $(1, i, 0)$, we have $W^\perp = \{v \mid \langle v, (1, i, 0) \rangle = 0\}$.

   Using the definition of inner product in $\mathbb{C}^3$, we have $\langle (z_1, z_2, z_3), (1, i, 0) \rangle = z_1 - iz_2$, so $W^\perp$ consists of all solutions to the equation $z_1 - iz_2 = 0$, which is equivalent to $z_1 = iz_2$.

   Hence $W^\perp$ consists of all vectors of the form (iz_2, z_2, z_3), or $z_2(i, 1, 0) + z_3(0, 0, 1)$, and a basis is $\{(i, 1, 0), (0, 0, 1)\}$.

   Notice that this makes sense: Both these vectors are orthogonal to $(1, i, 0)$, and they span a 2-dimensional subspace. Because $\mathbb{C}^3$ is 3-dimensional and W is 1-dimensional, we know $W^\perp$ must be 2-dimensional.

2. Suppose $\beta = \{v_1, v_2, v_3\}$ is an orthonormal basis for $\mathbb{R}^3$, and $v_1 = (\frac{3}{13}, \frac{4}{13}, \frac{12}{13})$. If the coordinates of $(1, 1, 1)$ in basis β are $[(1, 1, 1)]_\beta = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$, what is a?

   Since β is an orthonormal basis, we can always write $v = \langle v, v_1 \rangle v_1 + \langle v, v_2 \rangle v_2 + \langle v, v_3 \rangle v_3$. (Notice the order, $\langle v, v_i \rangle$. It matters if our field is $\mathbb{C}$.)

   Now $v = (1, 1, 1)$, and $a = \langle v, v_1 \rangle = \left\langle (1, 1, 1), (\frac{3}{13}, \frac{4}{13}, \frac{12}{13}) \right\rangle = \frac{19}{13}$.

3. If $T : \mathbb{C}^2 \to \mathbb{C}^2$ is defined by $T(z_1, z_2) = (z_1 + iz_2, z_1 - iz_2)$, what is $T^*(z_1, z_2)$?

   If $T = L_A$ then $T^* = L_{A^*}$.

   $A = \begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix}$ and so $A^* = \begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix}$. Therefore

   $T^*(z_1, z_2) = (z_1 + z_2, -iz_1 + iz_2)$.