Math 24
Winter 2010
Quiz 1 Sample Solutions

1. Give an equation (in any form you like) for the smallest subspace of $\mathbb{R}^3$ containing the vectors $(1, 1, 0)$ and $(1, -1, 0)$.

   Solution 1: Use the method from Section 1.1 in the textbook to get a vector parametric equation:
   $$\vec{x} = s(1, 1, 0) + t(1, -1, 0).$$

   Solution 2: Recall that the subspaces of $\mathbb{R}^3$ are the zero subspace, lines and planes through the origin, and all of $\mathbb{R}^3$. Since the given vectors do not lie on the same line through the origin, and are both in the xy-plane, the subspace they generate must be the xy-plane, which has scalar equation
   $$z = 0.$$

2. Complete this statement. If W is a subset of a vector space V, in order to verify that W (with the same operations as V) is a subspace of V, it is enough to check that:

   Solution:

   (a) The zero vector is in W. (Or, W is not the empty set.)
   (b) W is closed under addition.
   (c) W is closed under multiplication by scalars.

   Alternative, and equally acceptable, ways to say the same thing:

   W contains the zero vector (or, W is not the empty set) and is closed under addition and multiplication by scalars.

   (a) $\vec{0} \in W$ (or, $W \neq \emptyset$).
   (b) If $x \in W$ and $y \in W$, then $x + y \in W$.
   (c) If $x \in W$ and a is a scalar, then $ax \in W$.

   (a) $\vec{0} \in W$ (or, $W \neq \emptyset$).
   (b) $(x \in W \ \& \ y \in W) \implies x + y \in W$.
   (c) $(x \in W \ \& \ a \in F) \implies ax \in W$.

3. TRUE or FALSE? (You do not need to give reasons. If you do, you may in rare cases get limited partial credit for a wrong answer with a good reason.)

   (a) If F is any field, and x, y, and z are elements of a vector space V over F, then it is always the case that

   $$x + z = y + z \implies x = y.$$

   TRUE. (This is a theorem from the textbook, the cancellation law for vector addition.)

   (b) If V is a vector space over the rational numbers $\mathbb{Q}$, then V (with the same addition and scalar multiplication) can also be thought of as a vector space over the real numbers $\mathbb{R}$.

   FALSE. (An element of a vector space over $\mathbb{Q}$ cannot necessarily be multiplied by a real number not in $\mathbb{Q}$. For example, a nonzero matrix with rational entries, multiplied by $\sqrt{2}$ in the usual way, will not yield a matrix with rational entries.)

   (c) If V is a vector space over the real numbers $\mathbb{R}$, then V (with the same addition and scalar multiplication) can also be thought of as a vector space over the rational numbers $\mathbb{Q}$.

   TRUE. (This is because $\mathbb{Q} \subseteq \mathbb{R}$.)

   (d) If V is any vector space, the intersection of subspaces of V is always a subspace of V.

   TRUE. (This is a theorem from the textbook.)

   (e) The subset of $M_{2\times 2}(\mathbb{R})$ consisting of matrices whose trace is 1 is a subspace of $M_{2\times 2}(\mathbb{R})$. Recall that

   $$\textit{trace} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = a + d.$$

   FALSE. (Every subspace must contain the zero vector, and the trace of the zero matrix is not 1.)