Math 24
Winter 2010
Thursday, January 7

(1.) Which subsets of $\mathbb{R}^2$ are vector spaces over $\mathbb{R}$? Give a geometric description. (For example, the unit circle is not a vector space, because $(1, 0)$ lies on the unit circle, but $2(1, 0)$ does not. On the other hand, the x-axis is a vector space, because it is closed under addition and under multiplication by scalars.)

(2.) Which subsets of $\mathbb{R}^3$ are vector spaces?

(There is one more problem on the back.)

(3.) A vector space over the real numbers is sometimes called a real vector space. The real vector spaces $\mathbb{R}^2$, $\mathbb{C}$, $P_1(\mathbb{R})$ (polynomials over $\mathbb{R}$ of degree at most 1), and the space V of solutions to the differential equation $f'' + f = 0$ are very much alike:

$\mathbb{R}^2 = \{a(1,0) + b(0,1) \mid a \in \mathbb{R} \;\&\; b \in \mathbb{R}\}$;

$\mathbb{C} = \{a(1) + b(i) \mid a \in \mathbb{R} \;\&\; b \in \mathbb{R}\}$;

$P_1(\mathbb{R}) = \{a(1) + b(x) \mid a \in \mathbb{R} \;\&\; b \in \mathbb{R}\}$;

$V = \{a \sin x + b \cos x \mid a \in \mathbb{R} \;\&\; b \in \mathbb{R}\}$;

and in all cases, we add vectors, or multiply vectors by scalars, by adding, or multiplying, the numbers a and b. There is a sense in which these are the same vector space in different guises.

Describe the subsets of the vector spaces $\mathbb{C}$, $P_1(\mathbb{R})$, and V that are themselves vector spaces (with the same addition and scalar multiplication). (Use your answer to the first problem.)