Math 24

Winter 2010

Friday, February 26

For this problem, $V = \mathbb{R}^n$, and W is an m-dimensional subset of V. We define

$$W^{\perp} = \{v \mid w \cdot v = 0 \text{ for all } w \in W\},$$

where $\cdot$ denotes the familiar dot product.

For example, if $n = 3$ and $m = 2$, then the subspace W is a plane through the origin, and $W^{\perp}$ is the line through the origin perpendicular to that plane. If $n = 3$ and $m = 1$, then the subspace W is a line through the origin, and $W^{\perp}$ is the plane through the origin perpendicular to that line.

(1.) Show that $W^{\perp}$ is a subspace of V.

(2.) Suppose that $\beta = \{w_1, w_2, \ldots, w_m\}$ is a basis for W. Show that for any $v \in V$ we have

$$v \in W^{\perp} \iff w_i \cdot v = 0 \text{ for } i = 1, 2, \ldots, m.$$

(3.) Show that the dimension of $W^\perp$ is $n - m$.

(4.) Show that $V = W \oplus W^{\perp}$.