Math 24
Winter 2010
Monday, February 15

(1.) TRUE or FALSE?

(a.) The function $det : M_{n\times n}(F) \to F$ is a linear transformation.

(b.) The determinant of a $n \times n$ matrix is a linear function of each row of the matrix when the other rows are held fixed.

(c.) If $A \in M_{n\times n}(F)$ and $det(A) = 0$ then A is invertible.

(d.) If u and v are vectors in $\mathbb{R}^2$ emanating from the origin, then the area of the parallelogram having u and v as adjacent sides is $det \begin{pmatrix} u \\ v \end{pmatrix}$.

(e.) A coordinate system is right-handed if and only if its orientation equals 1.

(f.) The determinant of a square matrix can be evaluated by cofactor expansion along any row.

(g.) If two rows of a square matrix A are identical, then $det(A) = 0$.

(h.) If B is a matrix obtained from a square matrix A by multiplying a row of A by a scalar, then $det(B) = det(A)$.

(i.) If B is a matrix obtained from a square matrix A by interchanging any two rows, then $det(B) = -det(A)$.

(j.) If B is a matrix obtained from a square matrix A by adding k times row i to row j, then $det(B) = k\, det(A)$.

(k.) If $A \in M_{n\times n}(F)$ has rank n then $det(A) = 0$.

(l.) The determinant of an upper triangular matrix equals the product of its diagonal entries.

(2.) Evaluate the determinant of the following matrix, first by cofactor expansion along any row, second by using elementary row operations to transform it to an upper triangular matrix.

$$\begin{pmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ 2 & 3 & 0 \end{pmatrix}$$

For the remaining problems, let $G : M_{n\times n}(F) \to F$ be any function such that

(a.) G is a linear function of any row, when the other rows are held fixed.

(b.) If two rows of a matrix A are identical, then $G(A) = 0$.

(c.) $G(I_n) = 1$.

Show the following:

(3.) If B is obtained from A by multiplying row i by the scalar r, then $det(B) = r\, det(A)$. (Hint: Use the fact that G is a linear function of row i when the other rows are held fixed.)

(4.) If row i of A consists entirely of zeroes, then $det(A) = 0$.

(5.) If B is obtained from A by adding a scalar multiple of row i to row j, then $G(B) = G(A)$. (Hint: Use the fact that G is a linear function of row j when the other rows are held fixed.)

(6.) If B is obtained from A by interchanging row i and row j, then $det(B) = -det(A)$. (Hint: This type 1 elementary row operation can be accomplished by a combination of type 2 and type 3 operations.)

(7.) $G(A) = det(A)$ for any $n \times n$ matrix A. (Hint: A can be transformed by elementary row operations to either I_n or a matrix with a row of zeroes.)