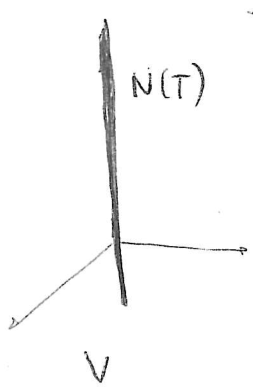
Definition Let $T: V \to W$ be a linear transformation. If $N(T)$ and $R(T)$ are finite dimensional, then

$$\text{nullity}(T) = \dim(N(T))$$
$$\text{rank}(T) = \dim(R(T)).$$

Theorem 9.2 (Dimension formula)
Let $T: V \to W$ be a linear transformation. If V is finite dimensional, then

$$\text{nullity}(T) + \text{rank}(T) = \dim V$$



Proof. Let $\{u_1, \dots, u_k\}$ be a basis of $N(T)$. Extend it to a basis of V, say $\{u_1, \dots, u_k, u_{k+1}, \dots, u_n\}$.
We need to show that a basis of $R(T)$ has the same number of vectors as in the set $\{u_{k+1}, \dots, u_n\}$.
Thus, it suffices to show that

$$S = \{T(u_{k+1}), \dots, T(u_n)\}$$

is a basis of $R(T)$.