Example Let $D: P(\mathbb{R}) \to P(\mathbb{R})$ be defined by

$$D(y) = 3y'' + 2y' - xy \qquad \text{where } y = y(x) \in P(\mathbb{R})$$

For example, if $y = x^3 + 5$, then

$$D(y) = D(x^3+5) = 3\cdot(x^3+5)'' + 2(x^3+5)' - x(x^3+5)$$

$$= 3(6x) + 6x^2 - x^4 - 5x$$

$$= -x^4 + 6x^2 + 13x$$

Is D linear?

Example Let $a, b \in \mathbb{R}$, $a < b$, and $T: C(\mathbb{R}) \to \mathbb{R}$ be defined by

continuous functions

$$T(f) = \int_a^b f(x)\,dx \qquad \text{for } f \in C(\mathbb{R})$$

Is T linear?

Example Let $S: C(\mathbb{R}) \to C(\mathbb{R})$ be defined by

$$S(f) = f^2 \qquad \text{for } f \in C(\mathbb{R})$$

Is f linear?