Linear Combinations (cont'd)

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Theorem. *The span of any subset S of a vector space V is a subspace of V. Moreover, any subspace of V that contains S must also contain span(S).*

- A subset S of a vector space V **generates** (or **spans**) V is $\text{span}(S) = V$. In this case we say that the vectors of S generate (or span) V.

Linear Dependence and Linear Independence

- A subset S of a vector space V is called **linearly dependent** if there exist a finite number of distinct vectors $u_1, u_2, \ldots, u_n$ in S and scalars $a_1, a_2, \ldots, a_n$, not all zero, such that

$$a_1 u_1 + a_2 u_2 + \ldots + a_n u_n = 0.$$

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- A subset S of V that is not linearly dependent is called **linearly dependent**. We say that the vectors of S are linearly independent.

Results about linear dependence and linear independence

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Theorem. *Let S be a linearly independent subset of a vector space V, and let v be a vector in V that is not in S. Then $S \bigcup \{v\}$ is linearly dependent if and only if $v \in$ span(S).*