Systems of Linear Equations

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Subspaces (cont'd)

Theorem. *Any intersection of subspaces of a vector space V is a subspace of V.*

Linear Combinations

- Let V be a vector space and S a nonempty subset of V. A vector v in V is called a **linear combination** of vectors of S if there exist a finite numbers of vectors $u_1, u_2, \ldots, u_n$ in S and scalars $a_1, a_2, \ldots, a_n$ in F such that

$$v = a_1 v_1 + a_2 v_2 + + a_n v_n.$$

- We say that v is a linear combination of the vectors $v_1, v_2, \ldots, v_n$.

- We call $a_1, a_2, \ldots, a_n$ the **coefficients** of the linear combination.

- Let S be a nonempty subset of a vector space V. The **span** of S, denoted span(S), is the set consisting of all linear combinations of the vectors in S. We define span$(\emptyset) = \{0\}$.

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Theorem. *The span of any subset S of a vector space V is a subspace of V. Moreover, any subspace of V that contains S must also contain span(S).*

- A subset S of a vector space V **generates** (or **spans**) V is span$(S) = V$. In this case we say that the vectors of S generate (or span) V.