Gram-Schmidt Orthogonalization Process

Lecture 25

March 7, 2007

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- A subset S of V is an orthonormal basis for V if it is an ordered basis that is orthonormal.

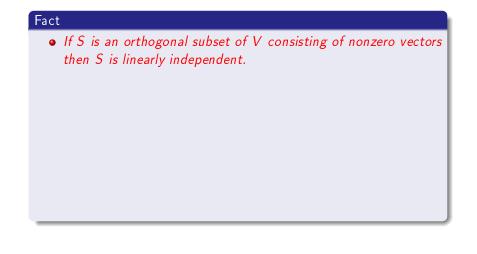
Why Study Orthogonal and Orthonormal Sets and Basis?



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Fact

- If S is an orthogonal subset of V consisting of nonzero vectors then S is linearly independent.
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- Any finite dimensional inner product space has an orthonormal basis.
- If $\beta = \{v_1, v_2, \dots, v_n\}$ is an orthonormal basis for V, then for any $x \in V$

$$x=\sum_{i=1}^n \langle x,v_i\rangle v_i.$$

The coefficients $\langle x, v_i \rangle$ are called the Fourier coefficients.

Theorem

Let V be an inner product space and $S = \{w_1, w_2, \ldots, w_n\}$ be a linearly independent subset of V. Define $S' = \{v_1, v_2, \ldots, v_n\}$, where $v_1 = w_1$ and

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Mathematical induction is used to prove that every statement in an infinite sequence of statements is true. It is done by

- proving that the first statement in the infinite sequence of statements is true, and then
- proving that if any one statement in the infinite sequence of statements is true, then so is the next one.

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The simplest and most common form of mathematical induction proves that a statement holds for all natural numbers n and consists of two steps:

- The basis: showing that the statement holds when n = 0.
- 2 The inductive step: showing that if the statement holds for n = m, then the same statement also holds for n = m + 1.

Mathematical Induction: The First Example

Example

Show that $1+2+3+\cdots+n=rac{n(n+1)}{2}.$

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Thank you and good luck! The End!

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