

Systems of Linear Equations

Computational Aspects – Gauss Elimination

Lecture 20

February 23, 2007

Equivalent Systems of Equations

Definition

Two systems of linear equations are called equivalent if they have the same solution set.

Theorem

Let $Ax = b$ be a system of m linear equations in n unknowns, and let C be an invertible $m \times m$ matrix. Then the system $(CA)x = Cb$ is equivalent to $Ax = b$.

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Corollary

Let $Ax = b$ be a system of m linear equations in n unknowns. If $(A'|b')$ is obtained from $(A|b)$ by a finite number of elementary row operations, then the system $A'x = b'$ is equivalent to the original system.

The Reduced Row Echelon Form

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- 3 The first nonzero entry in each row is 1 and it occurs in a column to the right of the first nonzero entry in the preceding row.

The Reduced Row Echelon Form

Theorem

Gaussian elimination transforms any matrix into its reduced row echelon form.