Vector Spaces

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Lecture 2

Vector Spaces

- A **vector space** (or **linear space**) V over a field F consists of a set on which two operations (called **addition** and **scalar multiplication**, respectively) are defined such that

  1. For all x, y in V, $x + y = y + x$.
  2. For all x, y, z in V, $(x + y) + z = x + (y + z)$.
  3. There exists an element in V denoted by 0 such that $x + 0 = x$ for each x in V.
  4. For each element x in V there exists and element y in V such that $x + y = 0$.
  5. For each element x in V, $1x = x$.
  6. For each pair of elements a, b in F and each element x in V, $(ab)x = a(bx)$.

7. For each element a in F and each pair of elements x, y in V, $a(x+y) = ax + ay$.
8. For each pair of elements a, b in F and each element x in V, $(a+b)x = ax + bx$.

7. For each element a in F and each pair of elements x, y in V, $a(x+y) = ax + ay$.
8. For each pair of elements a, b in F and each element x in V, $(a+b)x = ax + bx$.

- The elements of the field F are called **scalars.**

- The elements of the vector space V are called **vectors.**

Cancellation Law for Vector Addition

Theorem. *If* x, y, *and* z *are vectors in a vector space* V *such that* $x + z = y + z$, *then* $x = y$.

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- The vector 0 is unique and it is called the **zero vector**.
- The vector y is unique and it is called the **additive inverse** $-x$.

Properties of Scalar Multiplication

Theorem. *In any vector space V, the following statements are true:*

1. $0x = 0$ *for each* $x \in V$.

2. $(-a)x = -(ax) = a(-x)$ *for each* a *in* F *and each* x *in* V.

3. $a0 = 0$ *for each* $a \in F$.