Systems of Equations
Theoretical Aspects

Lecture 19

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Definition

Systems of Equations

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$$Ax = b.$$
- A **solution** to the system of equations is an n-tuple
$$s = \begin{pmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \end{pmatrix} \in F^n$$
such that $As = b$.

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- Otherwise the system is called nonhomogeneous.

Theorem

Let $Ax = 0$ be a homogeneous system of linear equations. Let K denoted the solutions set of $Ax = 0$. Then $K = N(L_A)$; Hence K is a subspace of F^n of dimension $n - rank(L_A) = n - rank(A)$.

Systems of Equations

Corollary

If $m < n$, the system $Ax = 0$ has a nonzero solution.

Theorem

Let K be the solution set of a system of linear equations $Ax = b$, and let $K_H$be the solution set of corresponding homogeneous system $Ax = 0$. Then for any solution s to $Ax = b$

$$K = \{s\} + K_H.$$

Theorem

Let $Ax = b$ be a system of n linear equations in n unknowns. If A is invertible, then the system has exactly one solution, namely $A^{-1}b$. Conversely, if the system has one solution, then A is invertible.

Theorem

Let $Ax = b$ be a system of linear equations. Then the system is consistent if and only if $rank(A) = rank(A|b)$.