Sets, Fields, and Vectors

January 10, 2007

Lecture 1

Sets

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- If x is an element of the set A we write $x \in A$. Otherwise we write $x \notin A$.
- Two sets are **equal** if they contain exactly the same elements.
- A set B is called a **subset** of a set A, written $B \subseteq A$ or $A \supseteq B$, if every element of B is an element of A.
- The empty set is denoted by $\emptyset$.

Operations with sets

- The **union** of two sets A and B is the set of elements that are in A, or B, or both:

$$A \bigcup B = \{x : x \in A \text{ or } x \in B\}.$$

- The **intersection** of two sets is the set of elements that are in both sets:

$$A \bigcap B = \{x : x \in A \text{ and } x \in B\}.$$

- If Λ is an index set and $\{A_\alpha : \alpha \in \Lambda\}$ is a collection of sets, the union of these sets is

$$\bigcup_{\alpha \in \Lambda} A_\alpha = \{x : x \in A_\alpha \text{ for some } \alpha \in \Lambda\},$$

  and the intersetion of these sets is

$$\bigcap_{\alpha \in \Lambda} A_\alpha = \{x : x \in A_\alpha \text{ for all } \alpha \in \Lambda\}.$$

Equivalence Relations

- A **relation** on a set A is a set S of ordered pairs of elements if A. We usually write $x \sim y$ in place of $(x, y) \in S$.

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- A relation S on A is called an **equivalence relation** on A if the following three conditions hold

  - For each $x \in A$, $x \sim x$. (reflexivity)
  - If $x \sim y$, then $y \sim x$. (symmetry)
  - If $x \sim y$ and $y \sim z$, then $x \sim z$ (transitivity).

Fields

- Roughly, a field is a set in which we can define an addition, a multiplication, a substraction, and a division.

- More precisely, a field F is a set on which two operations $+$ and $\cdot$ (called **addition** and **multiplication**, respectively) are defined so that the following conditions hold for all a, b, c in F:

1. $a + b = b + a$ and $a \cdot b = b \cdot a$
2. $(a + b) + c = a + (b + c)$ and $(a \cdot b) \cdot c = a \cdot (b \cdot c)$
3. There exist distinct elements 0 and 1 in F such that

$$0 + a = a \text{ and } 1 \cdot a = a$$

4. For each element a in F and each nonzero element b in F, there exist elements c and d in F such that

$$a + c = 0 \text{ and } b \cdot d = 1.$$

5. $a \cdot (b + c) = a \cdot b + a \cdot c.$

Substraction and Addition

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- *Substraction* is defined by

$$a - b = a + (-b).$$

- *Addition* is defined by

$$\frac{a}{b} = a \cdot b^{-1}.$$