Math 24
Lab 2
17 January 2002

Each group should write their solutions up together and turn in a single paper on Wednesday the 23rd. (There is no class on Monday the 21st.)

1. State the replacement theorem.

2. State the dimension theorem.

3. Suppose that $T : V \to V$ is linear and that $S = \{ v_1, \dots, v_n \}$ is a subset of V such that $\{ T(v_1), \dots, T(v_n) \}$ is linearly independent. Show that S is linearly independent.

4. Find an example of a linear map $T : \mathbf{R}^2 \to \mathbf{R}^2$ such that $N(T) = R(T)$.

5. Suppose that $\beta = \{ v_1, \dots, v_n \}$ is a basis for V and that $T : V \to V$ is linear. Prove that if T is one-to-one, then $\{ T(v_1), \dots, T(v_n) \}$ is a basis for V.