Your Name Goes Here

Math 24
Homework 8

#4.1.4 Let $A, B \in M_{m\times n}(\mathbb{C})$. Show that $\langle A, B\rangle_F = \sum_{j=1}^n \langle a_j, b_j\rangle$ and $\|A\|_F^2 = \sum_{j=1}^n \|a_j\|^2$.

#4.2.12c Carry out Gram-Schmidt on the following set of vectors:

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} \right\}$$

#4.2.17 Let $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \in M_3(\mathbb{R})$, and define a new inner product on $\mathbb{R}^3$ by $\langle x, y\rangle_A := \langle Ax, Ay\rangle$ where the inner product on the right-hand side is the standard inner product on $\mathbb{R}^3$. Find an orthonormal basis of $\mathbb{R}^3$ with respect to the $\langle \cdot, \cdot\rangle_A$.

#4.2.18 Let $A \in M_n(\mathbb{C})$ be an invertible matrix and define a non-standard inner product on $\mathbb{C}^n$ by $\langle x, y\rangle_A := \langle Ax, Ay\rangle$ where the inner product on the right-hand side is the standard inner product on $\mathbb{C}^n$. For the standard basis vectors $\{e_1, \dots, e_n\}$ for $\mathbb{C}^n$,

(a) Find $\langle e_j, e_k\rangle_A$.

(b) Under what circumstances is the standard basis of $\mathbb{C}^n$ orthonormal with respect to $\langle \cdot, \cdot\rangle_A$?

#4.3.2a Find the matrix (wrt the standard basis) of the orthogonal projection onto the span of the vectors $\left\{[1, 1, 1, 1]^T, [1, 2, 3, 4]^T\right\} \subset \mathbb{R}^4$.

#4.3.3b Find the matrix (wrt the standard basis) of the orthogonal projection onto the subspace

$$\{[x, y, z]^T \in \mathbb{R}^3 \mid 3x - y - 5z = 0\}.$$

Part 8 of Theorem 4.16 is probably of use.

#4.3.5a Find the point in the subspace U which is closest to the point x, where

$$U = \text{Span}\{[1, 0, -1, 2]^T, [2, -1, 1, 0]^T\} \text{ and } x = [1, 2, 3, 4]^T.$$

#4.3.5d Find the point in the subspace U which is closest to the point x, where

$$U = \ker \begin{bmatrix} 2 & 0 & -1 & 3 \\ 1 & 2 & 3 & 0 \end{bmatrix} \subseteq \mathbb{R}^4 \text{ and } x = [-2, 1, 1, 2]^T.$$

#4.3.7 Use simple linear regression to find the line in the plane which comes closest to the data points $(-2,1),(-1,2),(0,5),(1,4),(2,8)$.

#4.3.14 (a) Show that $V = \{A \in M_n(\mathbb{R}) \mid A^T = A\}$ and $W = \{A \in M_n(\mathbb{R}) \mid A^T = -A\}$ are subspaces of $M_n(\mathbb{R})$.

(b) Show that $V^\perp = W$ where the inner product is the usual Frobenius inner product.

(c) Show that for any $A \in M_n(\mathbb{R})$,

$$P_V(A) = \mathrm{Re}(A) \text{ and } P_W(A) = \mathrm{Im}(A),$$

where $\mathrm{Re}(A)$ and $\mathrm{Im}(A)$ are defined in Exercise 4.1.5 (ignore the i in the denominator of $\mathrm{Im}(A)$).