Your Name Goes Here

Math 24
Homework 5

#3.3.9 Suppose that $T \in \mathcal{L}(V, W)$, and that V is finite-dimensional. Prove that $\dim \operatorname{range}(T) \leq \dim V$.

#3.3.12 Suppose that $\dim V = n$ and that $T \in \mathcal{L}(V)$. Prove that T has at most n distinct eigenvalues.

#3.3.13 Suppose that $A \in M_n(\mathbb{F})$ has n distinct eigenvalues. Show there is a basis of $\mathbb{F}^n$ consisting of eigenvectors for A.

#3.4.6 Prove that if $T \in \mathcal{L}(V, W)$ and W is finite-dimensional, then T is surjective if and only if $\operatorname{rank}(T) = \dim W$.

#3.4.8 Prove that if $A \in M_{m\times n}(\mathbb{F})$ has rank r, then there exists $v_1 \ldots, v_r \in \mathbb{F}^m$ and $w_1, \ldots, w_r \in \mathbb{F}^n$ such that $A = \sum_{i=1}^r v_i w_i^T$. Author's hint: Write the columns of A as linear combinations of the basis $\{v_1, \ldots, v_r\}$ of $C(A)$. My hint: Do Quick Exercise #16 for insight.

#3.4.11 Suppose that $Ax = b$ is a 5×5 linear system which is consistent, but does not have a unique solution. Prove that there must be a $c \in \mathbb{F}^5$ so that the system $Ax = c$ is inconsistent.

#3.4.12 Suppose that, for a given $A \in M_3(\mathbb{R})$ there is a plane P passing through the origin in $\mathbb{R}^3$ such that the linear system $Ax = b$ is consistent if and only if $b \in P$. Prove that the set of solutions to the homogeneous system $Ax = 0$ is a line through the origin in $\mathbb{R}^3$.

#3.5.10 Let P be the plane

$$\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \middle| 4x + y - 2z = 0 \right\}.$$

(a) Find a basis for P.

(b) Determine whether each of the following vectors is in P, and if so give its coordinate representation in terms of your basis.

$$(i) \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad (ii) \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, \quad (iii) \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}.$$

#3.5.15 Show that the projection $P \in \mathcal{L}(\mathbb{R}^3)$ onto the xy-plane is diagonalizable.

#3.5.16 Let L be a line through the origin in $\mathbb{R}^2$, and let $P \in \mathcal{L}(\mathbb{R}^2)$ be the orthogonal projections onto L. (see Exercise 2.1.2.) Show that P is diagonalizable.