Your Name Goes Here

Math 24
Homework 3

#2.3.4 Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the map defined by first rotating counterclockwise by an angle θ and then reflecting across the line $y = x$. Find the matrix of T.

#2.3.11 Suppose that $A \in M_{m\times n}(\mathbb{F})$ is right-invertible, meaning that there is a matrix $B \in M_{n\times m}(\mathbb{F})$ so that $AB = I_m$. Show that $m \le n$. Author's hint: Show that given any vector $\mathbf{b} \in \mathbb{F}^m$, the $m \times n$ linear system $A\mathbf{x} = b$ is consistent, and use Theorem 1.2. There are other ways to reach the desired conclusion.

#2.4.4d,f Determine whether the matrices are singular or invertible; if invertible, find the inverse.

$$(d)\ \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad \text{and} \qquad (f)\ \begin{bmatrix} 1 & 0 & 2 & -1 \\ 0 & -1 & -2 & 1 \\ 2 & 0 & 1 & 0 \\ 0 & 2 & -3 & 2 \end{bmatrix}$$

#2.4.6a Compute $\begin{bmatrix} 1 & 0 & 1 \\ 3 & 2 & 0 \\ 0 & 1 & -2 \end{bmatrix}^{-1}$, and use it to solve the system

$$\begin{aligned} x + z &= 1 \\ 3x + 2y &= 2 \\ y - 2z &= 3 \end{aligned}$$

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#2.5.4d Determine whether the list of vectors $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$ span $\mathbb{R}^3$.

#2.5.6d Determine whether $\lambda = 3$ is an eigenvalue for $A = \begin{bmatrix} 7 & -2 & 4 \\ 0 & 1 & 0 \\ -6 & 2 & -3 \end{bmatrix}$, and if so express $\mathrm{Eig}_\lambda(A)$ as the span of some list of vectors.

#2.5.9 It is a fact (which you do not have to prove) the every solution to the homogeneous differential equation

$$\frac{d^2 f}{dt^2} + f(t) = 0$$

is a linear combination of $f_1(t) = \sin t$ and $f_2(t) = \cos t$.

(a) Show that $f_p(t) = 2e^{-t}$ is a solution to the differential equation

$$\frac{d^2 f}{dt^2} + f(t) = 4e^{-t}.$$

(b) Use Proposition 2.42 to show that every solution of the differential equation in part (a) is of the form

$$f(t) = 2e^{-t} + k_1 \sin t + k_2 \cos t$$

for constants $k_1, k_2 \in \mathbb{R}$.

(c) Determine all solutions $f(t)$ to the differential equation in part (a) which satisfy $f(0) = 1$.

(d) Determine all solutions $f(t)$ to the differential equation in part (a) which satisfy $f(0) = a$ and $f(\pi/2) = b$.

#2.5.10 Let $T : V \to W$ be a linear map. Prove that if U is a subspace of V, then $T(U)$ is a subspace of W.

#2.5.15 Suppose that $\lambda \in \mathbb{F}$ is an eigenvalue for $T \in \mathcal{L}(V)$, and $k \geq 1$ an integer. Show that λ^k is an eigenvalue for T^k, and $\mathrm{Eig}_\lambda(T) \subseteq \mathrm{Eig}_{\lambda^k}(T^k)$.

#2.5.16 Give an example of a linear map $T : V \to V$ with eigenvalue λ so that $\mathrm{Eig}_\lambda(T) \neq \mathrm{Eig}_{\lambda^2}(T^2)$.