Your Name Goes Here

Math 24

Homework 1

#1.2.4a Find the RREF of the matrix $\begin{bmatrix} 3 & 0 & 2 \\ 1 & -4 & 1 \end{bmatrix}$.

#1.2.4c Find the RREF of the matrix $\begin{bmatrix} -3 & 0 \\ 1 & 2 \\ 4 & -1 \end{bmatrix}$.

#1.2.6d Find all the solutions of the linear system

$$\begin{aligned} 3x + y - 2z &= -3 \\ x + 0y + 2z &= 4 \\ -x + 2y + 3z &= 1 \\ 2x - y + z &= -6 \end{aligned}$$

#1.2.6e Find all the solutions of the linear system

$$\begin{aligned} 3x + y - 2z &= -3 \\ x + 0y + 2z &= -4 \\ -x + 2y + 3z &= 1 \\ 2x - y + z &= -6 \end{aligned}$$

#1.2.13 Give examples of linear system of each of the following types if possible. Explain how you know they have the properties, or else explain why there is no such system.

(a) Underdetermined and inconsistent.

(b) Underdetrermined with a unique solution.

(c) Underdetermined with more than one solution.

(d) Overdetermined and inconsistent.

(e) Overdetrermined with a unique solution.

(f) Overdetermined with more than one solution.

(g) Square and inconsistent.

(h) Square with a unique solution.

(i) Square with more than one solution.

#1.3.10 Consider a linear system in vector form

$$x_1\mathbf{v}_1 + \cdots + x_n\mathbf{v}_n = \mathbf{b},$$

where $\mathbf{v}_1, \ldots, \mathbf{v}_n, \mathbf{b} \in \mathbb{R}^m$. Show that the system is consistent if and only if $\mathbf{b} \in \langle \mathbf{v}_1, \ldots, \mathbf{v}_n \rangle$.

#1.3.12 Let $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \in \mathbb{R}^3$, and suppose that the linear system $x\mathbf{v}_1 + y\mathbf{v}_2 + z\mathbf{v}_3 = \mathbf{0}$ has infinitely many solutions. Show that $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ lie in a plane in $\mathbb{R}^3$ which contains the origin $\mathbf{0}$.

#1.4.10 An $n \times n$ linear system over a field $\mathbb{F}$ is called **upper triangular** if the coefficient $a_{ij} = 0$ whenever $i > j$:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n &= b_1 \\ a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n &= b_2 \\ a_{33}x_3 + \cdots + a_{3n}x_n &= b_3 \\ \ddots \qquad\quad &\vdots \\ a_{nn}x_n &= b_n. \end{aligned}$$

Show that if $a_{ii} \neq 0$ for each i, then the system is consistent with a unique solution.

#1.5.4 Determine which of the following subsets are subspaces of $C[a,b]$. Here a, b are fixed real numbers and $a < c < b$.

(a) $V = \{f \in C[a,b] \mid f(c) = 0\}$.
(b) $V = \{f \in C[a,b] \mid f(c) = 1\}$.
(c) $V = \{f \in D[a,b] \mid f'(c) = 0\}$.
(d) $V = \{f \in D[a,b] \mid f' \text{ is constant}\}$.
(e) $V = \{f \in D[a,b] \mid f'(c) = 1\}$.

#1.5.11 Show that if U_1 and U_2 are subspaces of a vector space V, then

$$U_1 + U_2 := \{u_1 + u_2 \mid u_1 \in U_1, u_2 \in U_2\}$$

is also a subspace of V.