Eigenvectors_Diagaonalization

Enter the matrix by rows

```
A = matrix(QQ,[[-23,-8,-4,88,30],[-48,-13,0,144,48],[-37,-12,-5,134,45],
[-12,-4,0,39,12],[-1,0,-4,14,8]])
```

Check your entries to make sure they are right

```
A
```

$$\begin{pmatrix} -23 & -8 & -4 & 88 & 30 \\ -48 & -13 & 0 & 144 & 48 \\ -37 & -12 & -5 & 134 & 45 \\ -12 & -4 & 0 & 39 & 12 \\ -1 & 0 & -4 & 14 & 8 \end{pmatrix}$$

Characteristic polynomial and factored version

```
A.characteristic_polynomial();  A.characteristic_polynomial().factor()
```

$$x^5 - 6x^4 + 6x^3 + 16x^2 - 15x - 18$$

$$(x-2) \cdot (x-3)^2 \cdot (x+1)^2$$

The eigenvectors are lists as rows and normalized so leading entry is a 1

Format (eignvalue,[list of eigenvectors], algebraic multiplicity)

```
A.eigenvectors_right()
```

$$\left[\left(2, \left[\left(1, \frac{8}{3}, \frac{11}{6}, \frac{2}{3}, -\frac{1}{6}\right)\right], 1\right), \left(3, \left[(1, 0, 1, 0, 1), \left(0, 1, \frac{1}{3}, \frac{1}{3}, -\frac{2}{3}\right)\right], 2\right), \left(-1, \left[(1, 0, 2, 0, \right.\right.\right.$$

Gives the diagonal matrix and matrix of eigenvectors

```
[D,Q] = A.eigenmatrix_right();
```

See some results and conjugate by hand.

```
D; Q; Q.inverse()*A*Q
```

$$\begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 & 1 & 0 \\ \frac{8}{3} & 0 & 1 & 0 & 1 \\ \frac{11}{6} & 1 & \frac{1}{3} & 2 & -\frac{1}{4} \\ \frac{2}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{4} \\ -\frac{1}{6} & 1 & -\frac{2}{3} & 1 & -\frac{1}{2} \end{pmatrix}$$

$$\begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

To find eigenspace for eigenvalue 3 by hand, RREF of A - 3I

```
(A - 3*identity_matrix(5)).rref()
```

$$\begin{pmatrix} 1 & 0 & 0 & -2 & -1 \\ 0 & 1 & 0 & -3 & 0 \\ 0 & 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

But be careful. If the matrix is not diagonalizable, the routines give output, just junk.

See the example below

```
C = matrix(QQ,[[2,1],[0,2]])
```

```
C
```

$$\begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$$

```
C.eigenvectors_right()
```

$$[(2, [(1, 0)], 2)]$$

```
[D1,Q1]=C.eigenmatrix_right()
```

```
D1
```

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

```
Q1
```

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

```
Q1.is_invertible()
```

False