

Math 24
Spring 2012
Special Assignment due Monday, May 7

Let V be any vector space over F and W be a subspace of V . For any vector x in V , we defined the *coset* of W containing x to be

$$x + W = \{x + w \mid w \in W\}.$$

We denote the collection of cosets of W in V by V/W .

It turns out that V/W forms a vector space over F , with operations defined by

$$\begin{aligned}(x + W) + (y + W) &= (x + y) + W \\ a(x + W) &= (ax) + W.\end{aligned}$$

You may assume that this is true. (You proved part of this in the last two special homework assignments.)

Assignment: We can define a function T from V to V/W by $T(x) = x + W$.

Prove that T is a linear transformation.

Identify the null space and range of T .

If V is finite-dimensional, what can you conclude about the dimensions of V , W , and V/W ?