

Math 24
Spring 2012

Quiz Sample Solutions

Monday, May 14

1. (a) Write down a matrix B such that you can check whether the set $\{(1, 2, 3), (1, 0, 1), (2, 2, 1)\} \subseteq \mathbb{R}^3$ is linearly independent by finding the determinant of B .

$$\begin{pmatrix} 1 & 1 & 2 \\ 2 & 0 & 2 \\ 3 & 1 & 1 \end{pmatrix}$$

- (b) If the determinant of B is 6, is the set linearly independent?

YES

2. $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is determined by its action on the vectors of the basis $\beta = \{v_1, v_2, v_3\}$, as follows: $T(v_1) = 3v_1$, $T(v_2) = v_2 + v_1$, and $T(v_3) = v_3 - v_1$.

- (a) What is $[T]_\beta$?

$$\begin{pmatrix} 3 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- (b) If A is the matrix of T in the standard basis, what is $\det(A)$?

3

3. Find all eigenvalues of the matrix $\begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix}$.

$$\lambda_1 = 1 \quad \lambda_2 = -1$$

4. Find an eigenvector (any one eigenvector will do) for the matrix in (3).

$$(1, 1) \quad \text{or} \quad (0, 1)$$