

Math 24  
Spring 2012

Quiz Sample Solutions

Monday, May 7

1. What is the inverse of the matrix  $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix}$ ?  $\boxed{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{pmatrix}}$  (This is an elementary matrix.)
2. The matrix  $A$  is converted to the  $3 \times 3$  identity matrix by interchanging rows 2 and 3, adding row 1 to row 2, multiplying row 3 by  $\frac{1}{2}$ , and adding 5 times row 3 to row 1. What is the determinant of  $A$ ?  $\boxed{-2}$
3. If  $A$  is a  $4 \times 5$  matrix of rank 3, what is the dimension of  $\{\vec{x} \mid A\vec{x} = \vec{0}\}$ ?  $\boxed{2}$  (The rank of  $L_A$  is 3, and its domain has dimension 5, so the nullity of  $L_A$  is 2.)
4. Consider the following system of linear equations.

$$x_1 + x_2 + x_3 + x_4 + 4x_5 = 8$$

$$2x_1 + 2x_2 + 3x_3 + 1x_4 + 8x_5 = 16$$

$$-2x_1 - 2x_2 - 4x_5 = -8$$

$$x_1 + x_2 + 4x_3 + 4x_4 + 10x_5 = 20$$

The reduced row echelon form of the augmented matrix of the system is

$$\left( \begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 2 & 4 \\ 0 & 0 & 1 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

Give the complete solution of the system.

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \boxed{s \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} -2 \\ 0 \\ -1 \\ -1 \\ 1 \end{pmatrix} + \begin{pmatrix} 4 \\ 0 \\ 2 \\ 2 \\ 0 \end{pmatrix}}$$