MATH 24
GROUP PROBLEMS 1

Due Monday, April 5 at the beginning of class

Group Members Names:____

This a collection of problems designed to get a sense of what kinds of things you try when faced with a problem. What separates these problems from others you may have seen is that they are completely open ended. Do the best you can on each of them: I'm not looking for "right" and "wrong" answers, I'm just trying to see what you do with them.

Problem 1 A particular chocolate bar consists of a bunch of little rectangles all of the same size. The bar is m rectangles wide and n rectangles long. You want to eat a rectangle at a time so you break the big bar into the little ones. How many breaks do you make?

Problem 2 Let $p(n) = n^2 + n + 41$ where n is a positive integer. Is $p(n)$ always prime?

Problem 3 You have ten pennies arranged in a circle. Some show heads and some show tails. When you remove one showing heads the coins touching it (maybe none) and flipped. When is this game winnable?

Problem 4 What is wrong with this proof by induction:

Theorem. *All horses have the same color*

Proof. We prove this using mathematical induction. Clearly every horse in a set of one horse has the same color. This completes the base step. Now assume that any set of n horses are all the same color and see what happens when we add a horse. We label the horses with the numbers $1, 2, \dots, n, n+1$. By the induction hypothesis the horses $1, 2, \dots, n$ have the same color and so do the horses $2, 3, \dots, n+1$ since they are both sets of size n. Since both sets share horses $2, 3, \dots, n$ the $n+1$ horses are of the same color. $\square$

Problem 5 An unsolved problem in number theory is that there are an infinite number of pairs $(n, n+2)$ so that both n and $n+2$ are prime. E.g., $(3, 5)$, $(17, 19)$, etc. These pairs are called *twin primes*. How many triplet primes are there? I.e., triples $(n, n+2, n+4)$ where each entry is prime.