Math 23 Spring 2009 Second Midterm Exam

Instructor (circle one): Chernov, Sadykov

Wednesday May 13, 2009
6-8 PM Carpenter 013

PRINT NAME: ______________________________

Instructions: This is a closed book, closed notes exam. **Use of calculators is not permitted. You must justify all of your answers to receive credit.**

You have **two hours**. Do all the problems. Please do all your work on the paper provided.

The Honor Principle requires that you neither give nor receive any aid on this exam.

Grader's use only:

1. ________ /10

2. ________ /10

3. ________ /10

4. ________ /10

5. ________ /10

6. ________ /10

7. ________ /10

8. ________ /10

Total: ________ /80

1. **(a, 1 point)** Find the general solution of the homogeneous differential equation $y'' - 9y = 0$.

   (b, 6 points) Use the method of undetermined coefficients to find a particular solution of the differential equation $y'' - 9y = e^{3t} + t$.

   (c, 1 point) Find the general solution of $y'' - 9y = e^{3t} + t$.

   (d, 2 points) Solve the initial value problem $y'' - 9y = e^{3t} + t$ with $y(0) = 1$, $y'(0) = \frac{3}{54}$.

2. **(a, 8 points)** Use the Variation of Parameters to find a particular solution of the differential equation $y'' - 6y' + 9y = \dfrac{e^{3t}}{1+t^2}$.

(b, 2 points) Find the general solution of the equation $y'' - 9y' + y = \dfrac{e^{3t}}{1+t^2}$.

3. You are given a damped spring-mass system. The weight is equal to 128 lb (g=32), the spring constant is 1 lb/ft, and the damping coefficient is $\gamma \ \frac{\text{lb}\cdot\text{s}}{\text{ft}}$.

   (a, 3 points) Find γ so that the system is critically damped.

   (b, 4 points) Find the general form of the solution of the differential equation describing our spring-mass system with $\gamma = 4$.

   (c, 3 points) The system as above is set at rest and suddenly set in motion at $t = 0$ with initial velocity equal to 1 $\frac{\text{ft}}{\text{s}}$. Find the position of the mass at time $t = 1$.

4. **(a, 5 points)** Find the general solution of the homogeneous equation $y^{(6)} + y^{(4)} = 0$.

   (b, 3 points) Determine the suitable form for a particular solution $Y(t)$ in the method of undetermined coefficients in the equation $y^{(6)} + y^{(4)} = 3t + 5$. Do not attempt to find $Y(t)$ explicitly.

   (c, 2 points) Determine the suitable form for a particular solution $Y(t)$ in the method of undetermined coefficients in the equation $y^{(6)} + y^{(4)} = \cos(3t)$. Do not attempt to find $Y(t)$ explicitly.

5. **(a, 3 points)** Find the recurrence equation for coefficients of the series solution of

$$(4 - x^2)y'' + 2y = 0, \qquad x_0 = 0.$$

(b, 3 points) Find the first four terms in each of two solutions y_1, y_2 (unless the series terminates sooner).

(c, 4 points) Find the general term in each solution.

6. Determine $\phi''(\pi)$ and $\phi'''(\pi)$ if $y = \phi(x)$ is a solution of the initial value problem

$$x^2y'' + y' + (\cos(x))y = 0, \qquad y(\pi) = 2, \quad y'(\pi) = 0.$$

7. Determine the lower bound for the radius of convergence of a series solution about the given point x_0:

$$(x^3 - 1)y'' + xy' + 4y = 0.$$

(a, 5 points) $x_0 = 5$.

(b, 5 points) $x_0 = -5$.

8. Find all eigenvalues and eigenvectors of the given matrix:

$$A = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 5 & -1 \\ 0 & 3 & 1 \end{pmatrix}$$