Second Exam

MATH 23 — WINTER 2014

NAME:

SECTION: **11** **12**

This exam has 8 questions on 12 pages, for a total of 200 points.

You have 120 minutes to answer all questions.

This is a closed book exam.

Use of calculators and other electronic devices is not permitted.

Show all your work, justify all your answers.

Question	Points	Score
1	40	
2	25	
3	15	
4	30	
5	20	
6	20	
7	30	
8	20	
Total:	200	

40 1. Consider the spring-mass system for a mass $m = 1$ described by the equation:

$$u'' + 5u' + 6u = 3e^{-t}$$

(a) Find the transient solution.

(b) Find the steady state solution using the method variation of parameters.

(c) Suppose we want to change the damping so that the system is not critically damped or overdamped. If everything else remains the same what are the possible values of the damping constant?

25 2. Consider the spring-mass system from the previous problem.

(a) Write the equation we obtain if we change the damping constant and the external force to 0.

(b) Using this new equation and the initial conditions $u(0) = -1$, $u'(0) = 3\sqrt{2}$ find the period, frequency, amplitude, and phase of the solution. Reduce your answers as much as possible.

15 3. What is a lower bound on the radius of convergence of the power series solution to $(x^2+1)(x+1)y'' + xy' - 3y = 0$ centered at $x = 1$?

30 4. Let $y = \sum_{n=0}^{\infty} a_n x^n$ be a power series solution to the equation

$$y'' + x^2 y' + 2xy = 0$$

(a) Show that $a_2 = 0$ and that the coefficients satisfy $a_{n+3} = -\frac{a_n}{n+3}$ for all $n \geq 0$.

(b) Use the recurrence relation to find a power series solution to the initial value problem

$$y'' + x^2 y' + 2xy = 0, y(0) = 1, y'(0) = 0$$

Express your answer in summation notation.

(c) What is $y^{(101)}(0)$? Why?

20 5. An Euler equation $L[y] = 0$ has complex solution $y = x^{2+3i}$. Show how to obtain a fundamental set of real solutions to the equation. *You do not have to justify that your answer is a fundamental set.*

20 6. Find the general solution of $(x-2)^2y'' + 7(x-2)y' + 9y = 0$.

You should not have to use the method of power series to solve this.

30 7. Let $A = \begin{bmatrix} 2 & \alpha \\ 1 & 0 \end{bmatrix}$.

(a) Determine conditions on α which ensure the matrix A has

- two real eigenvalues
- no real eigenvalues

(b) In the case $\alpha = 3$, find two linearly independent eigenvectors of A.

20 8. (a) Solve the matrix equation:

$$\begin{pmatrix} 2 & -2 & -3 \\ 1 & 6 & 2 \\ 4 & 6 & -1 \end{pmatrix} \vec{v} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

(b) What does your solution tell you about the columns of the matrix $\begin{pmatrix} 2 & -2 & -3 \\ 1 & 6 & 2 \\ 4 & 6 & -1 \end{pmatrix}$?

Scrap paper