Math 23 Winter 2006

Differential Equations

First Midterm Exam

Monday, January 30, 6-8 PM
Bradley 101

Your name (please print): _____

Instructor: Chernov.

Instructions: This is a closed book, closed notes exam. **Use of calculators is not permitted**. You are allowed to bring one letter-size sheet of paper with any data you want written on it. **You must justify all of your answers to receive credit,** unless instructed otherwise in a given problem.

You have **two hours** to work on all **11** problems. The total score is the sum of your **10** best scores. Please do all your work in this exam booklet.

The Honor Principle requires that you neither give nor receive any aid on this exam.

Grader's use only

1. _____ /10

2. _____ /10

3. _____ /10

4. _____ /10

5. _____ /10

6. _____ /10

7. _____ /10

8. _____ /10

9. _____ /10

10. _____ /10

11. _____ /10

Total: _____ /100

(1) Find the general solution to the differential equation $y' = (y-4)x$. Solve the initial value problem $y(0) = 5$.

(2) Verify that $u(x, y) = \ln(x^2+y^2)$ is a solution of the differential equation $u_{xx}+u_{yy} = 0$.

(3) Find the integrating factor and use it to find the general solution of the differential equation $xdy + (x+1)ydx = 0$

(4) A 1000-liter tank originally contains 100 liters of pure water. A mixture containing the concentration of 0.1 kilograms of salt per liter enters the tank at the rate of 2 liters/minute. The well stirred solution leaves the tank at the rate of 1 liters/minute. Find the amount of salt in the tank 100 minutes into this process. You do not have to simplify the answer. (**Please be careful when setting up the differential equation.)**

(5) Solve the initial value problem. $y'' + 4y = 0$, $y(0) = 0, y'(0) = 2$.

(6) Find the longest interval in which the solution of the inital value problem

$$ty'' + e^t y' + \cos(t)y = \tan t,$$

$y(1) = 2006$, $y'(1) = 1$ is certain to exist. **Explain your answer.**

(7) If y_1, y_2 are two linearly independent solutions of $t^2y'' - ty' + \cos(t)y = 0$ and if $W(y_1, y_2)(1) = 1$, find the value of $W(y_1, y_2)(3)$.

(8) Find the general solution of the differential equation $y'' + 9y = \frac{1}{\sin 3t}$, $0 < t < \frac{\pi}{3}$.

(9) A spring-mass system has a mass of m kilogram and the spring constant equal to $4N/m$. The motion take place in the viscous fluid that offers a resistance numerically equal to 4 times the magnitude of the instantaneous velocity. Find the value of m so that the system is critically damped. **Explain your answer.**

(10) Use the method of reduction of order to find a second solution of the given differential equation: $t^2y'' + ty' - y = 0$, $y_1(t) = t$. **Show your work. Do not use some complicated general formulas.**

(11) Find the general solution of the differential equation $y'' + y' = 7 + e^t$.