Math 23 Diff Eq: In-class Midterm

65 minutes, 65 points. Answer all five questions, giving as much explanation as you have time for. No calculator needed; no algebra-capable ones allowed.

1. [10 points] Find the general solution to $ty' + 2y = 3t - 2$, for $t > 0$. Is the $t \to \infty$ behavior stable or unstable? To what, if any, function of t is this solution asymptotic?

2. [8 points] Find the *general* solution to $y'' + 4y' + 4y = t$

3. [21 points] Solve the following initial-value problems. In each case explain why your solution is the only solution, or find another solution (NB 4 points are reserved for this in each case so put in corresponding detail).

   (a) $y' = ty^{1/2}$ with $y(0) = 0$.

(b) $y^2 + (2xy + 1)y' = 0$ with $y(0) = 1$. (Remember to explain or find another solution as before...)

4. [10 points]

   (a) Solve the initial-value problem $y'' + y = \cos t$ with $y(0) = 0$ and $y'(0) = 0$. Note this is a driven mass-spring system released from rest.

   (b) What is the domain of t over which your solution is guaranteed to exist? (explain)

5. [16 points] Consider $y'' - x^2 y = 0$.

(a) Is $x_0 = 0$ a regular point? (explain your answer)

(b) Find the general power-series solution about $x_0 = 0$ writing the answer in the form $c_1 y_1(x) + c_2 y_2(x)$, where only the first 3 terms each of y_1 and y_2 need be given.

(c) Demonstrate that the y_1 and y_2 you found form a fundamental set of solutions.

(d) What is the most you can state about the radius of convergence of the series?