Summary of 3.2

Thm 3.2.1

Consider the IVP
$$\begin{cases} y'' + p(t)y' + q(t)y = g(t) \\ y(t_0) = y_0 \\ y'(t_0) = y_0' \end{cases}$$

If $p, q, g : (\alpha, \beta) \longrightarrow \mathbb{R}$ are continuous

Then there is exactly one solution defined in (α, β).

Now consider the homogeneous eqn

$$y'' + p(t)y' + q(t)y = 0 \qquad (*)$$

Thm 3.2.2

If y_1 and y_2 are solutions to $(*)$

then $c_1 y_1 + c_2 y_2$ are also solutions.

Thm 3.2.3

Every solution of the IVP
$$\begin{cases} (*) \\ y(t_0) = y_0 \\ y'(t_0) = y_0' \end{cases}$$

has the shape $c_1 y_1(t) + c_2 y_2(t)$ if

$$W(y_1, y_2)(t_0) \neq 0$$

Thm 3.2.4

If $p, q : (\alpha, \beta) \longrightarrow \mathbb{R}$ are continuous

and $W(Y_1, Y_2)(t_0) \neq 0$

then <u>every</u> solution of ⊛ in (α, β) has the

shape $c_1 Y_1(t) + c_2 Y_2(t)$.

Further, $W(Y_1, Y_2)(t) \neq 0$ for $t \in (\alpha, \beta)$.

Thm 3.2.7 Abel's thm

If $p, q : (\alpha, \beta) \longrightarrow \mathbb{R}$ are cont

then $W(Y_1, Y_2)(t) = c\, e^{-\int p(t)\, dt}$ for some c.

<u>Rmk</u> Thm 3.2.4 follows as an easy consequence of Abel's thm.

The upshot

To find general solution of ⊛ with p, q continuous, we must first find two solutions Y_1, Y_2 for which $W(Y_1, Y_2)(t_0) \neq 0$ for some $t_0 \in (\alpha, \beta)$.

Then every solution can be written as a linear combination of Y_1 and Y_2, ie,

$$Y(t) = c_1 Y_1(t) + c_2 Y_2(t) \quad \text{for } c_1, c_2 \in \mathbb{R}.$$