1. The total thermal energy at time t within a laterally insulated wire extending from $x = 0$ to $x = L$ is

$$E(t) = c \int_0^L u(x,t)\, dx$$

where $u(x,t)$ is the temperature function of the wire and c is a constant. If the wire is insulated at its ends, so that $u_x(0,t) = 0 = u_x(L,t)$, show that $E(t)$ is constant.

2. Find infinitely many solutions to the heat equation

$$\alpha^2 u_{xx} = u_t, \quad 0 < x < L, \quad t > 0$$

which satisfy the boundary conditions

$$u_x(0,t) = 0 = u_x(L,t), \quad t > 0.$$

(In doing this problem, you will have done problem 15 on page 547.)