# Math 23: Quiz

May 18, 2018

NAME: Solutions

SECTION (check one box) :

Section 1 (Kocyigit 11:30) ☐

Section 2 (Kocyigit 12:50) ☐

**Instructions:**

1. **Wait for signal** to begin.
2. **Write** your **name** in the space provided, and **check one box** to indicate which section of the course you belong to.
3. Please **turn off cell phones** or other electronic devices which may be disruptive.
4. Unless otherwise stated, you must **justify your solutions** to receive full credit. Work that is illegible may not be graded. Work that is scratched out will not be graded.
5. It is fine to leave your answer in a form such as ln(.02) or $\sqrt{239}$ or $(385)(13^3)$. However, if an expression can be easily simplified (such as $e^{\ln(.02)}$ or $\cos(\pi)$ or $(3-2)$), you should simplify it.
6. This exam is **closed book**. You may not use notes, or other external resource. You may use calculators. It is a violation of the honor code to give or receive help on this exam. However, you may ask the instructor for clarification on problems.
7. In True/False questions circle true, false or if you think the given information is not sufficient for a conclusion then write N/A and explain. Unless the question states "No explanation needed" you must explain your answer.
8. If you use a theorem you should check (and write clearly) whether the conditions of the theorem hold.

**Honor statement:** I have neither given nor received any help on this exam, and I attest that all of the answers are my own work.

**Signature:** ______________________

1. Given a homogeneous system

$$x' = P(t)x \qquad (H)$$

where $P(t)$ is a 2x2 matrix valued continuous function on $t \in \mathbb{R}$. Answer the below questions.

(a) How many solutions of (H) can satisfy the initial conditions $x(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $x'(0) = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$?

**Circle one:** (One or no solutions) (exactly one)

(none or infinitely many) (none)

2

**Explain:** $x' = P(t)x$, $x(0) = \binom{1}{2}$ has a unique soln. say $y(t)$.
If $\binom{3}{4}$ coincides with $y'(0)$ then ~~above~~ given problem has one soln. otherwise no soln.

(b) How many solutions of (H) can satisfy the initial conditions $x(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$?

**Circle one:** (One or no solutions) (exactly one)

(none or infinitely many) (infinitely many)

2

**Explain:** By uniqueness theorem since $P(t)$ is continuous.

(c) Suppose that $P(t)$ is a constant real matrix with an eigenvalue $2i$ and corresponding eigenvector $\begin{pmatrix} i \\ 1 \end{pmatrix}$.

Write the general solution of $(H)$. If it is not possible to write from the given information then explain why.

2

Answer: $y(t) = c_1\begin{pmatrix} -\sin 2t \\ \cos 2t \end{pmatrix} + c_2\begin{pmatrix} \cos 2t \\ \sin 2t \end{pmatrix}$ in $\mathbb{R}$

complex soln: $(\cos 2t + i\sin 2t)\begin{bmatrix} i \\ 1 \end{bmatrix} = \begin{pmatrix} -\sin 2t + i\cos 2t \\ \cos 2t + i\sin 2t \end{pmatrix} = \begin{pmatrix} -\sin 2t \\ \cos 2t \end{pmatrix} + i\begin{pmatrix} \cos 2t \\ \sin 2t \end{pmatrix}$

Note: also ok. to give gen soln in $\mathbb{C}$: $c_1 e^{2it}\binom{i}{1} + c_2 e^{-2it}\binom{-i}{1}$

(d) In part (c) if $P(t)$ is a constant **complex** valued matrix with an eigenvalue $2i$ and eigenvector $\begin{pmatrix} i \\ 1 \end{pmatrix}$ will your answer change? Explain why.

2

Answer: Yes. if $P(t)$ is complex it is not possible to write general soln. from the given information. Because eigenvalues / eigenvectors don't come in complex conjugate pairs.

2. Suppose $P(t)$ is a 2x2 matrix valued continuous function on $t \in \mathbb{R}$, and let $x^{(1)}$ be a solution to

$$x' = P(t)x + \begin{pmatrix} 1 \\ 2 \end{pmatrix} \qquad (1)$$

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such that $x^{(1)}(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$. Let $x^{(2)}$ and $x^{(3)}$ be solutions to the corresponding homogeneous system

$$x' = P(t)x \qquad (2)$$

such that $x^{(2)}(0) = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$ and $x^{(3)}(0) = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$. Find a solution $x$ to the non-homogeneous system (1) that satisfies $x(0) = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$ in terms of $x^{(1)}$, $x^{(2)}$, and $x^{(3)}$.

Answer: $X(t) = \frac{4}{3} X^{(2)} + \frac{6}{4} X^{(3)} + X^{(1)}$

gen soln: $X(t) = \alpha X^{(2)} + \beta X^{(3)} + X^{(1)}$

$\Rightarrow \begin{pmatrix} 4 \\ 6 \end{pmatrix} = X(0) = \alpha \begin{pmatrix} 3 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 4 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \alpha = 4/3$, $\beta = 6/4$

3. **Circle** true or false. If the statement can't be concluded from the given information write **N/A**. No explanation needed.

1

[T / F] If $y_1$ and $y_2$ are solutions of the equation

$$y' = cos(t^2)y + t$$

and if the graphs of $y_1$ and $y_2$ intersect at some point then $y_1(t) = y_2(t)$ for all $t$.

1

[T / F] There are infinitely many solutions of the equation

$$y'' + ty' + sin(t)y = e^{t^2}$$

whose graphs intersect each other at origin.

(see midterm solutions)