Math 23, Spring 2017

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Dartmouth College

Random stuff

Engineering Open House

Today April 7, 2017, 5:30–8:00pm

Thayer School of Engineering

Definition

An equation

$$M(x,y) + N(x,y)y' = 0$$

is called exact, if there is a ψ such that there exists a $\psi(x,y)$ such that

$$M = \frac{\partial \psi}{\partial x} \quad N = \frac{\partial \psi}{\partial y}.$$

Let $\phi(x)$ be a curve satisfying $\psi(x, \phi(x)) = C$ for some constant C. Then $y = \phi(x)$ is a solution to the original ODE.

- Check it!

Exact equations, Exercise 1

Definition

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Exercise 1

$$(2x + 3) + (2y - 2)y' = 0$$

Find M, N, and Ψ.

Main theorem

Theorem 2.6.1

Let the functions $M, N, M_y := \frac{\partial M}{\partial y}, N_x := \frac{\partial N}{\partial x}$ be continuous on the rectangular region $R = (\alpha, \beta) \times (\gamma, \delta)$. Then

$$M(x,y) + N(x,y)y' = 0$$

is exact on R if and only if

$$M_y(x,y) = N_x(x,y) \tag{1}$$

at all points (x,y) in R. In other words, there exists a function ψ such that

$$\psi_x(x,y) = M(x,y) \qquad \psi_y(x,y) = N(x,y) \tag{2}$$

if and only if (1) holds.

Exercise 1

Solve the IVP

$$(2x + 3) + (2y - 2)y' = 0, \quad y(1) = 3$$

Integration factors

- If $\dfrac{M_y - N_x}{N}$ only depends on x, then

$$\frac{\mu'(x)}{\mu(x)} = \frac{M_y - N_x}{N}$$

- If $\dfrac{N_x - M_y}{M}$ only depends on y, then

$$\frac{\mu'(y)}{\mu(y)} = \frac{N_x - M_y}{M}$$

Exercise 30

Solve the ODE

$$4\frac{x^3}{y^2} + 3\frac{1}{y} + \left(3\frac{x}{y^2} + 4y\right)\frac{\mathrm{d}y}{\mathrm{d}x} = 0$$

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Exercise 26

Solve the ODE

$$y' = e^{2x} + y - 1$$

Generic integration factors

- Exercise 24 covers $\mu = \mu(xy)$
- What if $\mu = \mu(x^2 y^3)$?

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  $\rightsquigarrow \dfrac{\mu'(x^2y^3)}{\mu(x^2y^3)} = \dfrac{N_x - M_y}{M3x^2y^2 - N2xy^3}$ must be a function of x^2y^3

Generic integration factors

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- What if $\mu = \mu(x^2y^3)$?
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- Solve the ODE

$$4xy^2 + 5x^2yy' = 0$$