Math 23, Spring 2017

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Exercise 2.4.32

Solve the IVP:

$$y' + 2y = g(t) = \begin{cases} 1, & 0 \le t \le 1 \\ 0, & t > 1 \end{cases}, \quad y(0) = 0$$

- One can think that someone pulled a switch at $t = 1$

1. Solve $y_1' + 2y_1 = 0, \quad y_1(0) = 0$
2. Solve $y_2\prime + 2y_2 = 1, \quad y_2(1) = y_1(1)$
3. Then $y(t) = \begin{cases} y_1(t) & 0 \le t \le 1 \\ y_2(t) & t > 1 \end{cases}$ is a solution for the IVP, even though $y'(t)$ is not continuous.

Three important steps in modeling

- Construction of the model
- Analysis of the model
- Comparison with experiment or observation

Exercise 2.3.1

Consider a tank used in certain hydrodynamic experiments. After one experiment the tank contains 200 L of a dye solution with a concentration of 1 g/L. To prepare for the next experiment, the tank is to be rinsed with fresh water flowing in at a rate of 2 L/min, the well-stirred solution flowing out at the same rate. Find the time that will elapse before the concentration of dye in the tank reaches 1% of its original value.

Exercise 2.3.1 – Spin-off # 1

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Spin-off # 1

Now assume that the fluid coming in has $0.5g/L$ how does this change the behaviour?

Exercise 2.3.1 – Spin-off # 2

Spin-off # 2

Now assume there are 2 tubes coming in

- $0.5g/L$ at the $2L/min$
- $0.1g/L$ at the rate $1L/min$

There is only one tube coming out at the rate $2L/min$. The tank's max capacity is $300L$, and initially it has $200L$ of solution with concentration $1g/L$. Find the mass of dye at the moment when the tank overflows.

Compound Interest, Exercise 9

Exercise 2.3.9

A certain college graduate borrows \$8000 to buy a car. The lender charges interest at an annual rate of 10%. Assuming that interest is compounded continuously and that the borrower makes payments continuously at a constant annual rate k, determine the payment rate k that is required to pay off the loan in 3 years. Also determine how much interest is paid during the 3-year period.