Math 23, Spring 2017

Edgar Costa

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Dartmouth College

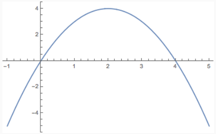
Random stuff

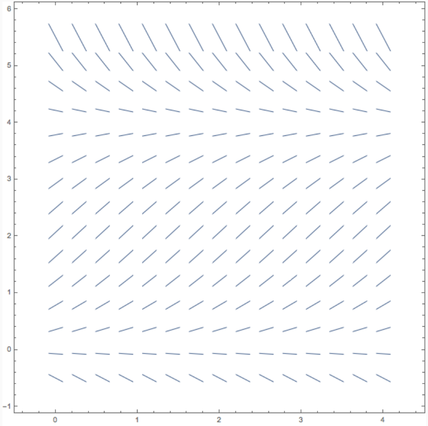
- Tutorials: Su, Tu, Th : 7 pm - 9 pm
- Homework: Due Wednesday’s 3:30 pm
    - first one due next week
    - on the boxes on the 1st floor
- Office hours:
    - Costa’s: M 4-5:30 ; W 4-5
    - Gelb’s: M 2:30-4; F 2:30-3:30
- I won’t be around today for my Office Hours
- Today
    - §1.3 Classification of Differential Equations
    - §2.1 Solve linear ordinary DEs

Exercise 1.2.11

Draw the direction field of $y' = y(4 - y)$.

$$\frac{dy}{dt} \text{ vs } y$$





$$y = \frac{4e^{4t}}{D + e^{4t}} \qquad \text{(general solution)}$$

Classification of Differential Equations

Some vocabulary so we can describe different situations that arise

- **Ordinary** Differential Equations vs **Partial** Differential Equations
- What is a system of Differential Equations?
- **Order** of a Differential Equation
- **Linear** vs Nonlinear
- What is a solution

Ordinary vs Partial

Definition

An **ordinary differential equation** (ODE) is a differential equation containing one or more functions of one independent variable and its derivatives.

A **partial differential equation** (PDE) is a differential equation that contains unknown multivariable functions and their partial derivatives.

If the function in the DE depends on just one variable $\Rightarrow$ ODE. We have only considered ODEs so far.

- $\dfrac{dv(t)}{dt} = 10 - \dfrac{v(t)}{5}$ is an ODE, $v(t)$ only depends on t.
- $a^2 \dfrac{\partial^2 u(x,t)}{\partial x^2} = \dfrac{\partial^2 u(x,t)}{\partial t^2}$ is a PDE, $u(x,t)$ depends on x and t.

System

We have only considered a single differential equation, sometimes there are models that involves several functions, all connected by DEs, known as **systems of DEs**.

For example,

$$\frac{dx}{dt} = ax - \alpha xy$$
$$\frac{dy}{dt} = -cy + \gamma xy$$

Order

Definition

The **order** of a differential equation is the order of the highest derivative that appears in the equation.

- First order ODE: $\frac{dy}{dt} + \frac{2}{t}y = 4t$
- Third order ODE: $y''' + 2e^t y'' + y'y = t^4$
- More generally, an n^{th} order ODE can always be put in the form

$$F(t, y, y', \dots, y^{(n)}) = 0$$

for some function F. (Like degree of a polynomial.)

Linear vs nonlinear

Definition

An ordinary differential equation

$$F(t, y, y', \dots, y^{(n)}) = 0$$

is **linear** if F is a linear function of the variables $y, y', \dots, y^{(n)}$.

- $y' = y^2$ is nonlinear because y appears to the 2nd power.
- $y' = \sin y$ is also nonlinear. $\sin y$ is not a linear function in y.
- $y' = ay + b$ is a linear ODE.
- In general linear ODE of order n is

$$a_n(t)y^{(n)} + \cdots + a_1(t)y' + a_0(t)y = g(t)\,.$$

An ODE not of this form is called **nonlinear**.

Solutions

Definition

A **solution** to an ODE $F(t, y, \ldots, y^{(n)}) = 0$ on an interval (a, b) is a function $y(t)$ that satisfies this equation (i.e., that makes it true) for all t in (a, b).

- It's important to keep track of the interval of definition of a solution!
- Q: Do all DEs have solutions?
  A: No, not necessarily. A silly example is $(y')^2 + 1 = 0$; this has no solution because $(y')^2 = -1$ is impossible in $\mathbb{R}$.
- Q: Can a DE have more than one solution?
  A: We have seen that this is often the case, since many solutions involve an arbitrary constant C, known as **general solutions**. We will talk more about "how many" solutions a DE can have later in the course.

Example 1

Solve

$$(4 + t^2)\frac{dy}{dt} + 2ty = 4t$$

Exercise 13

Solve the Initial Value Problem (IVP)

$$y' - y = 2te^{2t}, \quad y(0) = 1$$

Solving linear ODEs

Exercise 16

Solve the IVP

$$y' + \frac{2}{t}y = \frac{\cos(t)}{t^2}, \quad y(\pi) = 0(t > 0)$$

Exercise 7

Solve

$$y' + 2ty = 2te^{-t^2}$$