$\underline{Ex}$. $y + (2t - ye^y)\frac{dy}{dt} = 0$ (from Apr 2)

$\left.\begin{array}{l} M_y = 1 \\ N_t = 2 \end{array}\right\}$ not equal so not exact.

Check for an integrating factor which depends on y only.

- Check if $\frac{N_t - M_y}{M}$ depends on y only.

$$\frac{N_t - M_y}{M} = \frac{2-1}{y} = \frac{1}{y} \checkmark$$

- Solve $\mu' = \frac{N_t - M_y}{M}\mu \implies \frac{d\mu}{dy} = \frac{1}{y}\mu$ separable

$$\frac{1}{\mu}d\mu = \frac{1}{y}dy$$

$$\ln|\mu| = \ln|y|$$

$\mu = y$ is our integrating factor! (Depends only on y)

$\underline{\text{Next}}$: Multiply by $\mu = y$.

$y^2 + (2ty - y^2e^y)\frac{dy}{dt} = 0$. Solving this is equivalent to solving oringinal problem.

Is it exact? $\left.\begin{array}{l} M_y = 2y \\ N_t = 2t \end{array}\right\}$ equal $\checkmark$

now solve . . .

Want to find $\psi(t,y)$ s.t.

$$\psi_t = M$$
$$\psi_y = N.$$

① We know $\psi_t = M = y^2$

So ψ must look like $\psi = ty^2 + h(y)$ (*)

② From this, we can see that ψ_y must be

$$\psi_y = 2ty + h'(y)$$

③ We know $\psi_y = N$, so let's compare to find $h'(y)$.

$$\psi_y = 2ty + h'(y) = N = 2ty - y^2e^y$$

Therefore $h'(y) = -y^2e^y$,

Solve for $h(y)$ using Integration-by-parts (twice)

Find $h(y) = -e^y(y^2 - 2y + 1)$

④ Plug back into (*) to get

$$\psi = ty^2 - e^y(y^2 - 2y + 1)$$

⑤ Solutions of our DE are of form:

$$ty^2 - e^y(y^2 - 2y + 1) = c$$

for constant c.