Solutions to Practice Problems for Math 23 Final

Chapter 7:

1. $\mathbf{x} = c_1 e^{2t}\begin{pmatrix}1\\1\end{pmatrix} + c_2\left[te^{2t}\begin{pmatrix}1\\1\end{pmatrix} + e^{2t}\begin{pmatrix}1\\0\end{pmatrix}\right]$

Chapter 9:

2. The critical points are $(0,0)$ and $(2,3)$. The point $(0,0)$ is an asymptotically stable node. The point $(2,3)$ is an unstable saddle point.

Chapter 5:

3. (a) $x_0 = 1$ is regular singular, $x_0 = -2$ is singular but not regular, $x_0 = 0$ is ordinary.
   (b) $x_0 = 1$ and $x_0 = 0$ are both regular singular, $x_0 = -2$ is ordinary.
4. (a) $y_1 = 1 - \frac{1}{2}x^2 - \frac{1}{6}x^3 + \frac{1}{24}x^4 + \cdots$ and $y_2 = x - \frac{1}{3}x^3 - \frac{1}{6}x^4 + \cdots$
   (b) $y_1 = x - \frac{1}{4}x^3 + \frac{1}{48}x^5 + \cdots$ and $y_2 = -\ln(x)y_1 + x^{-1}(1 - \frac{3}{16}x^4 + \frac{7}{288}x^6 + \cdots)$
5. For part (a) of the previous problem:
   (a) The Wronskian is 1.
   (b) Lower bound is 1.
   (c) $y_1 + 2y_2$

Chapter 10:

6. $f(x) = \dfrac{1}{2} + \dfrac{1}{\pi n}\displaystyle\sum_{n=1}^{\infty}\sin(2n\pi x)$

8. $u(x,t) = \dfrac{20}{\pi}\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}(1-(-1)^n)\sin(nx)e^{2n^2t}$

9. $u(x,t) = \dfrac{8}{\pi^2 n^2}\displaystyle\sum_{n=1}^{\infty}\sin(n\pi x/4)\left[\frac{8}{\pi n}((-1)^n - 1)\cos(n\pi t/4) + (1-(-1)^n)\sin(n\pi t/4)\right]$