HOMEWORK 2

MATH 23, SPRING 2007

This homework set is due Monday April 9, 2007 at the beginning of class.

Problems from Boyce and di Prima. For each problem, please show all of your work in a coherent manner. Explain your methodology in complete sentances.

- 2.1 : $\#1, 8$
- 2.2 : $\#9, 16, 21$
- 2.4 : $\#3, 7$
- 2.6 : $\#2, 4, 13$
- 2.7 : Find a numerical solution to $y' = 1 + t - y,\ y(0) = 2$ via Euler method. To do this, use the following matlab code (euler.m from the course webpage).

```
% The Euler method, (c) L. Euler 1768.
% Code from A. Barnett 9/28/05
% Roughly follows algorithm on p.  104 of the text
f = @(t,y) 1+t-y; % set up function f(t,y)
t0 = 0; y0 = 2; % Initial conditions
h = 0.1; % time step
T = 4; % final (stopping) time

N = (T-t0)/h; % number of steps
ys(1) = y0; % first y,t given by IC
ts(1) = t0; % (NB indexing starts at 1)
for n=1:N
ys(n+1) = ys(n) + h*f(ts(n),ys(n)); % Euler update
for y
ts(n+1) = ts(n) + h; % fill the time values too
end

% you now have ys containing a vector of y values
% corresponding to t values in the vector ts
```

To modify this code, you may with to look at commands from intro.m. For this problem, you should:

(1) Plot a graph of the numerical solution using $+$ signs. On the same plot, add a graph (using solid lines) of the exact solution. Label the axes.

(2) Next, plot the difference between the numerical and the exact solution. What is the magnitude of the largest error you see?

(3) Repeat these steps two times with $h = 0.01, h = 0.001$. Roughly, by what factor do errors shrink? Using this data, estimate how big N needs to be to ensure error of less than 10^{-6}.