Math 23, Spring 2007
Lecture 16

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Outline

Material from last class

- First order systems of equations
- Predator-Prey model
- Translating higher order ODE into first order systems

Existence and Uniqueness

Theorem

Given a system of first order equations

$$x_1' = F_1(t,\vec{x}),\ x_2' = F_2(t,\vec{x}),\ldots,x_k' = F_k(t,\vec{x})$$

If F_i and $\frac{dF_i}{dx_j}$ are continuous on a region R defined by $\alpha < t < \beta, \alpha_i < x_i < \beta_i$ for all i,j and $(t_0, x_1^0, \ldots, x_k^0)$ is a point in R, then there is an interval $t_0 - h < t < t_0 + h$ in which there is a unique solution to the system above subject to the initial conditions

$$x_1(t_0) = x_1^0, \ldots, x_k(t_0) = x_k^0$$

Eigenvalues and Eigenvectors

Let A be a $k \times k$ matrix.

Definition

An eigenvalue of A is a number λ with the property that there exists a vector v so that

$$Av = \lambda v$$

Such a vector v is called an eigenvector associated to λ

Finding eigenvalues

$$Av = \lambda v \iff (A - \lambda I)v = 0$$

Theorem

The equation $Mv = 0$ has a nonzero solution if and only if $det\ M = 0$.

To find eigenvalues, solve $det(A - \lambda I) = 0$ for λ.

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Example

Find the eigenvalues for

$$\begin{pmatrix} 3 & 2 & 2 \\ 1 & 4 & 1 \\ -2 & -4 & -1 \end{pmatrix}$$

Finding eigenvectors

To find the associated eigenvector for a given λ solve the system

$$(A - \lambda I)\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

for x_1, x_2

Examples

Find the eigenvalues and eigenvectors for

$$\begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ 3 & 2 & 1 \end{pmatrix}$$

Work for next class

- Read 7.5
- Homework 6 is due Monday 5/7/07