# Math 23, Spring 2007 

 Lecture 15
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## 4/30/07

## Outline

## Midterm results

Last class

Last class
Today's material
Series solutions around ordinary points
Series solutions around
ordinary points
Next class

Today's material
Series solutions around ordinary points
Linear systems of equations
Linear systems of equations

Next class

## Midterm results



Figure: Histogram from total score

- In class: mean $=31$, std $=8$
- Take home: mean $=35$, std=9
- Total: mean $=66$, std $=15$


## Material from last class

- Series solutions for second order linear ODE

$$
y=\sum_{n=0}^{\infty} a_{n}\left(t-t_{0}\right)^{n}
$$

- Further examples


## Example from last class

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Midterm results

## Last class

Today's material
Series solutions around
ordinary points
Series solutions around
ordinary points
Legendre's equation:

$$
\left(1-x^{2}\right) y^{\prime \prime}-2 x y^{\prime}+\alpha(\alpha+1) y=0
$$

## Example from midterm

In the second problem on the take home midterm, you were asked to transform a second order equation

$$
a y^{\prime \prime}+b y^{\prime}+c y=g(t)
$$

## Last class

Today's material
Series solutions around
ordinary points
Series solutions around
ordinary points
into two (coupled) first order equations:

$$
\begin{aligned}
& L_{1}[y(t)]=u(t) \\
& L_{2}[u(t)]=g(t)
\end{aligned}
$$

Point: one can then solve two first order equations (often simple) rather than a single, potentially harder higher order equation.

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## Examples of systems

## Predator/Prey

The Lotka-Volterra model:

$$
\begin{aligned}
& \frac{d H}{d t}=a_{1} H-b_{1} H P \\
& \frac{d P}{d t}=-a_{2} P+b_{2} H P
\end{aligned}
$$

where $H, P$ are the two populations, $a_{1}$ is the birth rate of $H, a_{2}$ is the death rate of $P$ and $b_{1}, b_{2}$ are the coefficients of the interaction between predator and prey HP.

## Transformation of second order systems

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Midterm results
Last class
Today's material
Series solutions around
ordinary points
Series solutions around

$$
y^{\prime \prime}+p(t) y^{\prime}+q(t) y=g(t)
$$

## Transformation of second order systems

$$
y^{\prime \prime}+p(t) y^{\prime}+q(t) y=g(t)
$$

Let $u=y^{\prime}, v=y$. Then this system becomes

$$
\begin{aligned}
& v^{\prime}=u \\
& u^{\prime}=g(t)-p(t) u-q(t) v
\end{aligned}
$$

## Using matlab

Consider a difficult second order system:
Today's material
Series solutions around ordinary points
Series solutions around

$$
y^{\prime \prime}+\sin (y)=0
$$

## We can using ode 45 to solve this system. See linsys.m on our website.

## Using matlab

Math 23, Spring

Consider a difficult second order system:

$$
y^{\prime \prime}+\sin (y)=0
$$

Convert it to a first order system:

$$
u=v^{\prime}, u^{\prime}=-\sin (v)
$$

## We can using ode 45 to solve this system. See linsys.m on our website.

## Using matlab

Consider a difficult second order system:

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## Work for next class

- Read: 7.1-5.4
- Homework 5 is due wednesday $5 / 1$, Homework 6 is posted today and due Monday 5/7/07

