Math 23, Spring 2007 Lecture 15

Scott Pauls 1

¹Department of Mathematics Dartmouth College

4/30/07

◆□▶ ◆□▶ ◆ 臣▶ ◆ 臣▶ 三臣 - のへで

Math 23, Spring 2007

Scott Pauls

Midterm results

Last class

Today's material Series solutions around ordinary points Series solutions around ordinary points

Outline

Midterm results

Last class

Today's material

Series solutions around ordinary points Linear systems of equations Linear systems of equations

Next class

Math 23, Spring 2007

Scott Pauls

Midterm results

_ast class

Today's material Series solutions around ordinary points Series solutions around ordinary points

lext class

▲□▶ ▲□▶ ▲□▶ ▲□▶ ■ のの⊙

Midterm results

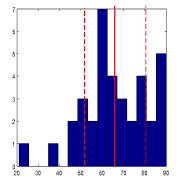


Figure: Histogram from total score

▲ロト ▲周 ト ▲ ヨ ト ▲ ヨ ト つのの

- In class: mean =31, std = 8
- Take home: mean =35, std=9
- Total: mean = 66, std = 15

Math 23, Spring 2007

Scott Pauls

Midterm results

Last class

Today's material Series solutions around ordinary points Series solutions around ordinary points

Material from last class

Series solutions for second order linear ODE

$$y=\sum_{n=0}^{\infty}a_n(t-t_0)^n$$

▲□▶ ▲□▶ ▲ □▶ ▲ □▶ ▲ □ ● ● のへで

Further examples

►

Math 23, Spring 2007

Scott Pauls

Midterm results

Last class

Today's material Series solutions around ordinary points Series solutions around ordinary points

Example from last class

Legendre's equation:

$$(1 - x^2)y'' - 2xy' + \alpha(\alpha + 1)y = 0$$

Math 23, Spring 2007

Scott Pauls

Midterm results

Last class

Today's material Series solutions around ordinary points Series solutions around ordinary points

Vext class

◆□▶ ◆□▶ ◆ □▶ ★ □▶ = □ ● ○ ○ ○

Example from midterm

In the second problem on the take home midterm, you were asked to transform a second order equation

$$ay'' + by' + cy = g(t)$$

into two (coupled) first order equations:

 $L_1[y(t)] = u(t)$

 $L_2[u(t)] = g(t)$

Point: one can then solve two first order equations (often simple) rather than a single, potentially harder higher order equation.

Math 23, Spring 2007

Scott Pauls

Midterm results

_ast class

Today's material Series solutions around ordinary points Series solutions around ordinary points

Next class

Example from midterm

In the second problem on the take home midterm, you were asked to transform a second order equation

$$ay'' + by' + cy = g(t)$$

into two (coupled) first order equations:

 $L_1[y(t)] = u(t)$

 $L_2[u(t)] = g(t)$

Point: one can then solve two first order equations (often simple) rather than a single, potentially harder higher order equation.

Math 23, Spring 2007

Scott Pauls

Midterm results

_ast class

Today's material Series solutions around ordinary points Series solutions around ordinary points

Next class

Examples of systems

Predator/Prey

The Lotka-Volterra model:

$$rac{dH}{dt} = a_1H - b_1HP$$

 $rac{dP}{dt} = -a_2P + b_2HP$

where H, P are the two populations, a_1 is the birth rate of H, a_2 is the death rate of P and b_1, b_2 are the coefficients of the interaction between predator and prey HP.

▲□▶ ▲□▶ ▲□▶ ▲□▶ ■ のの⊙

Math 23, Spring 2007

Scott Pauls

Midterm results

_ast class

Today's material Series solutions around ordinary points Series solutions around ordinary points

Transformation of second order systems

$$y'' + p(t)y' + q(t)y = g(t)$$

.

$$v' = u$$

$$u' = g(t) - p(t)u - q(t)v$$

▲□▶▲□▶▲□▶▲□▶ □ のQ@

Math 23, Spring 2007

Scott Pauls

Midterm results

Last class

Today's material Series solutions around ordinary points

Transformation of second order systems

$$y'' + p(t)y' + q(t)y = g(t)$$

Let u = y', v = y. Then this system becomes

$$v' = u$$

 $u' = g(t) - p(t)u - q(t)v$

▲□▶▲□▶▲□▶▲□▶ □ のQ@

Math 23, Spring 2007

Scott Pauls

Midterm results

Last class

Today's material Series solutions around ordinary points Series solutions around ordinary points

Using matlab

Consider a difficult second order system:

$$y'' + \sin(y) = 0$$

Convert it to a first order system:

$$u = v', u' = -\sin(v)$$

▲□▶ ▲□▶ ▲□▶ ▲□▶ ■ のの⊙

We can using ode45 to solve this system. See linsys.m on our website.

Math 23, Spring 2007

Scott Pauls

Midterm results

Last class

Today's material Series solutions around ordinary points Series solutions around ordinary points

Using matlab

Consider a difficult second order system:

$$y'' + \sin(y) = 0$$

Convert it to a first order system:

$$u=v', u'=-\sin(v)$$

▲□▶ ▲□▶ ▲□▶ ▲□▶ ■ のの⊙

We can using ode45 to solve this system. See linsys.m on our website.

Math 23, Spring 2007

Scott Pauls

Midterm results

Last class

Today's material Series solutions around ordinary points Series solutions around ordinary points

Using matlab

Consider a difficult second order system:

$$y'' + \sin(y) = 0$$

Convert it to a first order system:

$$u=v', u'=-\sin(v)$$

▲□▶ ▲□▶ ▲□▶ ▲□▶ ■ のの⊙

We can using ode45 to solve this system. See linsys.m on our website.

Math 23, Spring 2007

Scott Pauls

Midterm results

Last class

Today's material Series solutions around ordinary points Series solutions around ordinary points

Work for next class

- Read: 7.1-5.4
- Homework 5 is due wednesday 5/1, Homework 6 is posted today and due Monday 5/7/07

▲ロト ▲周 ト ▲ ヨ ト ▲ ヨ ト つのの

Math 23, Spring 2007

Scott Pauls

Midterm results

Last class

Today's material Series solutions around ordinary points Series solutions around ordinary points

Next class