

**MATH 23 - DIFFERENTIAL EQUATIONS, WINTER 2011**  
**FINAL**

PRINT NAME:

- (1) Find the first 3 nonzero terms of a series solutions corresponding to the larger root of the indicial equation at the regular singular point  $x=0$  of the differential equation:

$$2xy'' + y' + xy = 0$$

- (2) Consider the equation of heat conduction on a rod of length 40 with the temperature of both ends held at 0:

$$u_{xx} = u_t$$

Find the first three nonzero terms of the Fourier series for  $u$ , given that  $u(x, 0) = f(x)$  and  $u'(x, 0) = 0$  where  $f(x) = 50$  if  $10 < x < 30$  and  $f(x) = 0$  otherwise.

(3) Consider the system of equations:

$$x' = ax + 2y,$$

$$y' = -2x$$

- a. For what values of  $a$  is the equilibrium at  $(0, 0)$  stable?
- b. For what values is it unstable?
- c. For what values do solutions oscillate?
- d. Find the general solution to the system of equations in terms of real valued functions when  $a = 2$ .

- (4) Find all critical points of the following nonlinear system. For each critical point, linearize the system, find the eigenvalues, and indicate whether the equilibrium is stable or unstable.

$$x'(t) = xy(2 + x)$$

$$y'(t) = (7 - x)(y + x)$$