

**MATH 23 - DIFFERENTIAL EQUATIONS, SPRING 2011**  
**FINAL**

PRINT NAME:

SECTION:

- (1) A. Find the first 3 nonzero terms of series solutions corresponding to the larger root of the indicial equation at the regular singular point  $x=0$  of the differential equation:

$$x^2y'' + xy' + (x^2 - 1)y = 0$$

- B. extra credit: This is Bessel's equation of order 1. What does it have to do with the wave equation? (No more than 2 sentences, please!)

- (2) Consider the equation of a vibrating string of length 10 with fixed ends at 0 and 10:

$$u_{xx} = u_{tt}$$

Find the first three nonzero terms of the Fourier series for  $u$ , given that  $u(x, 0) = 0$  and  $u'(x, 0) = f(x)$  where  $f(x) = 1$  if  $1 < x < 9$  and  $f(x) = 0$  otherwise.

(3) Consider the system of equations:

$$x' = -2x + By,$$

$$y' = x + y$$

A. For what values of  $B$  does the solution have decreasing oscillations?

B. Find the general solution to the system of equations in terms of real valued functions when  $B = 25/4$ .

- (4) Find all critical points of the following nonlinear system. For each critical point, linearize the system, find the eigenvalues, and indicate whether the equilibrium is stable or unstable.

$$\begin{aligned}x'(t) &= xy - 8 \\y'(t) &= (x + 2)(y - 4)\end{aligned}$$