Math 23 WORKSHEET : Variation of Parameters.

Barnett
10/17/05

[3.7-10] Find a particular solution to $y'' - 2y' + y = \frac{e^t}{1+t^2}$

Find roots of char. eqn :

So lin. indep. homogeneous solutions are : $y_1(t) =$
$y_2(t) =$

So $W(t) = y_1 y_2' - y_2 y_1' =$

$$-\int \frac{y_2(s)\, g(s)}{W(s)}\, ds =$$

so parameter $u(t) =$

$$\int \frac{y_1(s)\, g(s)}{W(s)}\, ds =$$

NB: $\int \frac{ds}{1+s^2} = \tan^{-1} s + c$

so parameter $v(t) =$

Put it all together:

$$Y(t) = u(t)\, y_1(t) + v(t)\, y_2(t)$$

$$=$$

(If time) write general solution :

Math 23 WORKSHEET : Variation of Parameters. Barnett 10/13/05

[3.7.10] Find a particular solution to $y'' - 2y' + y = \frac{e^t}{1+t^2}$

Find roots of char. eqn : $r^2 - 2r + 1 = 0$

$(r-1)^2$ $r = +1$ twice

So lin. indep. homogeneous solutions are : $y_1(t) = e^t$

$y_2(t) = te^t$

So $W(t) = y_1 y_2' - y_2 y_1' = e^t(e^t + te^t) - te^t(e^t)$

$= e^{2t}.$

$$-\int^t \frac{y_2(s)\, g(s)}{W(s)}\, ds = \int^t \frac{se^s}{e^{2s}} \frac{e^s}{1+s^2}\, ds = \int^t \frac{s}{1+s^2}\, ds = \tfrac{1}{2}\ln|1+t^2|$$

$+C$ gets absorbed into homog. gen. soln. anyway

so parameter $u(t) = \frac{1}{2}\ln|1+t^2|$

$$\int^t \frac{y_1(s)\, g(s)}{W(s)}\, ds = \int^t \frac{e^s}{e^{2s}} \frac{e^s}{1+s^2}\, ds$$

$\boxed{\text{NB: } \int \frac{ds}{1+s^2} = \tan^{-1} s + C}$

so parameter $v(t) = \tan^{-1} t$

Put it all together:

$$Y(t) = u(t)\, y_1(t) + v(t)\, y_2(t)$$

$$= \tfrac{1}{2}\ln(1+t^2)\, e^t + (\tan^{-1} t)\, te^t$$

If time) write general solution : the above $+ c_1 e^t + c_2 te^t$