

# MATH 23 WORKSHEET : Undetermined Coefficients

10/17/07  
Bumelt

Consider  $y'' + 3y' + 2y = g(t)$  roots are what? : driving

i) for  $g(t) = e^{-3t}$ , guess the form of  $Y(t)$  and solve for coefficient:

$$y'' + 3y' + 2y = g(t)$$

$Y'' =$        $Y' =$        $\xleftarrow{d/dt} Y = ?$

ii) for  $g(t) = e^{-t}$ , guess the form of  $Y(t)$  and try as above:

What's gone wrong? Why?

Guess a better  $Y(t)$  form [hint: think back to repeated roots ...]

$$Y(t) = A \dots$$

Try it:

$$y'' + 3y' + 2y = e^{-t}$$

$Y'' =$        $Y' =$        $\xleftarrow{d/dt} Y =$

Solve for  $A$ :

iii) Bonus: solve  $y'' + 2y' + y = e^{-t}$

SOLUTIONS

MATH 23 WORKSHEET : Undetermined Coefficients

10/17/02  
Bunnell

roots are what? : -1, -2 watch out!

Consider  $y'' + 3y' + 2y = g(t)$  driving

i) for  $g(t) = e^{-3t}$ , guess the form of  $Y(t)$  and solve for coefficient

$$y'' + 3y' + 2y = e^{-3t}$$

$Y'' = 9Ae^{-3t}$       $Y' = -3Ae^{-3t}$       $Y = Ae^{-3t}$

$\text{coeffs in } e^{-3t} : 9A - 9A + 2A = 1 \quad A = 1/2$

so  $Y(t) = \frac{1}{2} e^{-3t}$  is a particular solution

ii) for  $g(t) = e^{-t}$ , guess the form of  $Y(t)$  and try as above:

$$Y'' = Ae^{-t} \quad Y' = -Ae^{-t} \quad Y = Ae^{-t}$$

$$A - 3A + 2A = 1 \quad \text{ie } 0A = 1$$

What's gone wrong? Why?

$e^{-t}$  was a solution to the homogeneous eqn.  $y'' + 3y' + 2y = 0$ , so LHS vanished so no use in matching the driving.

Note:  $e^{-2t}$  would also cause problem.

Guess a better  $Y(t)$  form [hint: think back to repeated roots ...]

$$Y(t) = Ate^{-t}$$

Try it:

$$y'' + 3y' + 2y = e^{-t}$$

$$Y'' = -Ae^{-t} - Ate^{-t} \quad Y' = Ae^{-t} - Ate^{-t} \quad Y = Ate^{-t}$$

Solve for A: coeffs of  $te^{-t}$  are:  $A - 3A + 2A = 0$  tells you nothing, but is consistent  
 coeffs of  $e^{-t}$ :  $-2A + 3A = 1$  so  $A=1$  ✓

ii) Bonus: solve  $y'' + 2y' + y = e^{-t}$ : since  $e^{-t}$  to  $t^2$  already hom. soln. use  $Y = t^2 e^{-t}$