MATH 23 WORKSHEET : Airy's series solutions

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Airy's Equation $\underbrace{y''}\ - \underbrace{xy}\ = 0$

set $y = \sum_0^\infty a_n x^n$

you do the same

... and shift index so power of x is n

... you shift so power of x is n!

$$\sum_0^\infty (n+2)(n+1)a_{n+2}x^n \quad - \quad\quad\quad = 0$$

Now equate powers of x, see what eqns get:

x^0 (const term) : so $a_2 =$

x^1 : so $a_3 =$

x^2 : so $a_4 =$

x^3 : etc.

x^4 :

x^5 :

Explain the pattern to your neighbor!

Which two coeffs control (linearly scale) all other terms in series?

Write series expansions for the two linearly-indep solns:

$y_1(x) = a_0\left[1 + \frac{x^{?}}{?} + \text{—} + \right.$

$y_2(x) = a_{?}\left[\quad + \right.$

Try to write the nth term in each series:

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10/19/05
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~ SOLUTIONS ~

Airy's Equation $y'' - xy = 0$

set $y = \sum_0^\infty a_n x^n$

... and shift index so power of x is n

you do the same

$x \sum a_n x^n = \sum_0 a_n x^{n+1}$

... you shift so power of x is n!

$$\sum_0^\infty (n+2)(n+1)a_{n+2}x^n \quad - \quad \sum_1 a_{n-1}x^n = 0$$

note doesn't contribute for $n=0$.

Now equate powers of x, see what eqns get:

x^0 (const term) : $2\cdot 1 a_2 = 0$ so $a_2 = 0$.

x^1 : $3\cdot 2\, a_3 = a_0$ so $a_3 = \frac{1}{3\cdot 2} a_0$

x^2 : $4\cdot 3 a_4 = a_1$ so $a_4 = \frac{a_1}{4\cdot 3}$

x^3 : $5\cdot 4\, a_5 = a_2$ etc. $a_5 = \frac{a_2}{\ } = 0$

x^4 : $6\cdot 5\, a_6 = a_3$ $a_6 = \frac{a_3}{6\cdot 5} = \frac{1}{6\cdot 5\cdot 3\cdot 2} a_0$

x^5 : $7\cdot 6\, a_7 = a_4$ $a_7 = \frac{a_4}{7\cdot 6} = \frac{a_1}{7\cdot 6\cdot 4\cdot 3}$

etc.. Explain the pattern to your neighbor!

Which two coeffs control (linearly scale) all other terms in series?

$a_0 \quad a_1 \qquad (a_{3n+2} = 0$ for $n = 0, 1, \ldots$

Write series expansions for the two linearly-indep solns:

$$y_1(x) = a_0 \left[1 + \frac{x^3}{3\cdot 2} + \frac{x^6}{6\cdot 5\cdot 3\cdot 2} + \frac{x^9}{9\cdot 8\cdot 6\cdot 5\cdot 3\cdot 2} \right.$$

? ?

$$y_2(x) = a_1 \left[x + \frac{x^4}{4\cdot 3} + \frac{x^7}{7\cdot 6\cdot 4\cdot 3} + \cdots \right.$$

?

don't worry - it's hard

Try to write the nth term in each series: $\frac{x^{3n}}{3n(3n-1)\cdot(3n-3)(3n-4)\cdots 3\cdot 2}$