Math 23 Diff Eq: Quiz 1 (1^{st}-order ODEs)

25 minutes, 25 points. Answer all questions, giving as much explanation as you have time for. No calculator needed; no algebra-capable ones allowed.

1. [8 points] Consider $y' = \sqrt{y}$

   (a) Solve it for $t > 0$ with the initial conditions $y(0) = 1$. Is the solution unique and if so can you say in what domain? (use a relevant theorem)

   (b) Solve it for $t > 0$ with the initial conditions $y(0) = 0$. Is the solution unique and if so can you say in what domain? (use a relevant theorem)

2. [8 points] A quantity $y(t)$ obeys $t^2 y' + 3ty = \dfrac{\cos t}{t}$ for $t > 0$.

   (a) Find the general solution.

(b) Find the solution with initial condition $y(\pi) = 1$.

(c) In what domain of t must the solution exist and be unique?

3. [9 points] Determine if each of the following equations is exact. If so, find the general solution $y(x)$, explicitly if possible (rather than implicitly).

(a)

$$2x \sin y + y + (-x^2 \cos y + y)\frac{dy}{dx} = 0$$

(b)

$$3x^2 - 2xy + 2 + (-x^2 + 3)\frac{dy}{dx} = 0$$

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